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Destabilization analysis of overlapping underground chambers induced by blasting vibration with catastrophe theory

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Abstract: According to the main characters of overlapping underground chambers, the roof (floor) of two adjacent underground chambers is simplified to the mechanical model that is the beam with build-in ends. And vibration load due to blasting is simplified to harmonic wave. The catastrophic model of double cusp for underground chambers destabilization induced by blasting vibration has been established under the circumstances of considering deadweight of the beam, and the condition of destabilization has been worked out. The critical safety thickness of the roof (floor) of underground chambers has been confirmed according to the destabilization condition. The influence of amplitude and frequency of blasting vibration load on the critical safety thickness has been analyzed, and the quantitative relation between velocity, frequency of blasting vibration and critical safety thickness has been determined. Research results show that the destabilization of underground chambers is not only dependent on the amplitude and frequency of blasting vibration load, but also related to deadweight load and intrinsic attribute. It is accordant to testing results and some related latest research results of blasting seismic effect. With increasing amplitude, the critical safety thickness of underground chambers decreases gradually. And the possibility of underground chambers destabilization increases. When the frequency of blasting vibration is equal to or very close to the frequency of beam, resonance effect will take place in the system. Then the critical safety thickness will turn to zero, underground chambers will be damaged severely, and its loading capacity will lose on the whole.

Key words: blasting vibration; overlapping underground chambers; destabilization; catastrophe theory; critical safety thickness

1 Introduction

At present, boring and blasting construction methods are mostly used in underground engineering, for example excavation in mine, building of tunnels and subways, large underground power houses and so on. However, the damage blasting operation brings to underground chamber (chambers) , such as shock wave, seismic wave, slingshots and blasting noise, can not be ignored, especially the seismic wave[1, 2]. Blasting vibration not only influences the safety of ground buildings, but also damages and disturbs the surrounding rock of underground chamber and adjacent chamber, even can induce the destabilization of underground chamber (chambers)[3−6]. Blasting vibration has become an important factor that affects the stability and induces the destabilization of underground chamber (chambers). The researches about stability and destabilization of underground chamber under blasting vibration are mostly situ tests. Regression analysis is carried out according to test data, and damage degree to underground chamber (chambers) blasting vibration brings is estimated by referring some relative rules[2−4]. What is more, TAN et al[5] and LIU et al[6] carry out the numerical simulation with dynamic finite element methods, and discuss the influence which blasting vibration brings to underground chamber. Based on looking on plenty relative research information, it is found that the researches about stability and destabilization of underground chamber (chambers) due to blasting vibration dynamic load are mainly for single chamber. The researches about complex underground chambers are comparatively few.

Actually, the destabilization of underground chamber (chambers) induced by blasting vibration is a sudden break phenomenon. And it has obvious nonlinear and discontinuous property. The catastrophe theory found by R.THOM is an effective tool to study the discontinuous phenomenon[7]. In the latest two decades, the applica-

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tion of the catastrophe theory in rock mass mechanics and engineering has made a great progress. PAN et al[8] and PAN[9] report that some rock mechanics and engineering problems are explained with catastrophe theory, for instance rockburst, rock fracturing process etc. QIN et al[10−13] analyze the mechanism by which external disturbance induces the rock mass destabilization of slope. At the hand of structure dynamic destabilization, SHEN and WEI[14] constitute the catastrophe model of elastic arch vibration destabilization ZUO et al[15] study the problem of rock fracture and destabilization under dynamic and static combined load with catastrophe theory. Researches about stability and destabilization of underground chamber (chambers) under blasting vibration dynamic load with catastrophe theory have not ever been reported.

The non-linear dynamic mechanism of underground chambers destabilization induced by blasting vibration is studied with catastrophe theory in this study. The catastrophic model of double cusp for underground chambers destabilization induced by blasting vibration has been established under the circumstances of considering deadweight of the beam, and the condition of destabilization and critical safety thickness of the roof (floor) of underground chambers have been worked out. The influence of amplitude and frequency of blasting vibration load on the critical safety thickness has been analyzed. It is significant for further understanding of the mechanism of underground chambers destabilization induced by blasting vibration.

2 Mechanical models

The original shape for the research in this study is large span underground chambers made up of two adjacent chambers. Its geological model is shown in Fig.1.

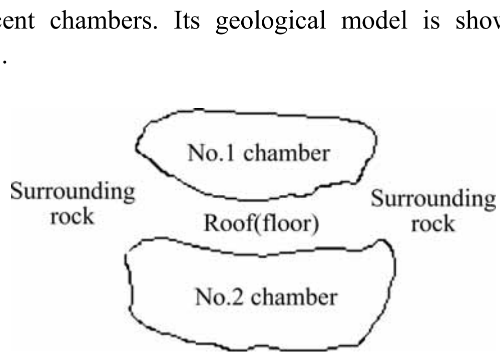

Fig.1 Geological model of adjacent chambers

Under the circumstance that the strength of surrounding rock is higher and its integrity is better, the roof (floor) can be simplified to a beam model with build-in ends, which is shown in Fig.2.

In Fig.2, $P(x, t)$ is the vertical blasting vibration dynamic load applied on the beam; P_0 is the deadweight of the beam, $P_0=mg$, m is the mass of the beam, g is the acceleration of gravity; q is the level field stress; L is the span of the beam; h is the thickness of the beam (or height).

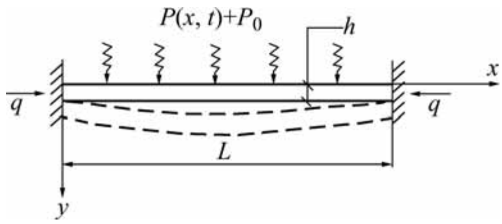

Fig.2 Simplified mechanical model

In order to establish the vibration equilibrium equation, the infinitesimal element dx of the beam is taken for analysis, which is shown in Fig.3.

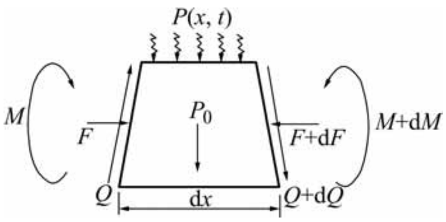

Fig.3 Infinitesimal element

Considering dynamic balance of the infinitesimal element in y direction, the following equations can be established according to d'ALEMBERT principle,

$$\frac{\partial Q}{\partial x} + P(x,t) + P_0 = \rho A \frac{\partial^2 \omega}{\partial t^2} \tag{1}$$

According to moment equilibrium,

$$\frac{\partial M}{\partial x} + F\frac{\partial \omega}{\partial x} = Q \tag{2}$$

where ρ is the density; A is the area of the cross-section of the beam. F, Q and M are the horizontal internal force, vertical internal force and bending moment at the cross-section of the beam respectively; ω is the deflection of the beam.

According to EULER-BERNOULLI hypothesis[16],

$$EI(\omega'''' - \omega_0'''') - \frac{EA}{2L}\omega''\int_0^L\left[(\omega_0')^2 - (\omega')^2\right]\mathrm{d}x + c\dot{\omega} + P(x,t) + P_0 + \rho\ddot{\omega} = 0 \tag{3}$$

where E, I are the modulus and the moment of inertia of the beam respectively; c is the damping coefficient, and $c>0$; ω_0 is the initial deflection of the beam, that is the displacement under the deadweight of the beam.

3 Catastrophic model

According to Ref.[14], $P(x, t)$ can be simplified to

$$P(x,t) = P'\cos\omega' t\sin\frac{\pi x}{L} \tag{4}$$

where P' is the amplitude value of blasting vibration dynamic load; ω' is the frequency value of blasting vibration dynamic load.

So, it can be assumed,

$$\omega(x,t) = y(t)\sin\frac{\pi x}{L} \tag{5}$$

Substituting Equation (5) into Equation (3) gives

$$\ddot{y} + k_0\dot{y} + r_0 y + \alpha y^3 + P'\cos\omega' t + P_0 = 0 \tag{6}$$

where $k_0=c/\rho>0$, $r_0=\pi^4 EI/(\rho L^4)$, $r_0 = \omega_0'^2$, and ω_0' is the natural frequency of the beam when the deadweight is not considered; $\alpha=\pi^4 EA/(4\rho L^4)$, α represents the nonlinear coefficient of the structure. When the value of α changes, the dynamic response of structure will be different. If $\alpha>0$, the stiffness of structure will be strengthened with increasing distortion, and the structure is gradual hard spring; If $\alpha=0$, it is linear vibration; If $\alpha<0$, the compliance of structure will be strengthened as the distortion increases, and the structure is gradual soft spring[13].

When k_0 and α are very little, and the distortion of the beam is little, Eqn.(6) is weak nonlinear, and its solution is approximate to linear equations. So the solution of Eqn.(6) can be supposed to

$$y(t) = H\cos(\omega' t + \phi) + H_0 \tag{7}$$

where H_0 is the amplitude induced by the deadweight of the beam, H_0 is assumed to be constant; H is the amplitude of dynamic response; ϕ is the response lag induced by damping c, and it is obvious that $\phi=0$ when c=0.

Substituting Eqn.(7) into Eqn.(6), and neglecting high order little quantity and third harmonic terms, and letting the coefficients of the first harmonic terms be equal, then after coordination, we have

$$\mathrm{tg}\phi = \frac{k_0\omega'}{\omega'^2 - r_0 - 3\alpha H^2/4 - 3\alpha H_0^2} \tag{8}$$

$$H^2\left(r_0 - \omega'^2 + 3\alpha H_0^2 + 3\alpha H^2/4\right)^2 + k_0^2\omega'^2 H^2 = P'^2 \tag{9}$$

Carrying out diffeomorphism transformation[7] for Eqn.(9), and canceling quadratic term about H^2 yields

$$(B+D)^3 + u(B+D) + v = 0 \tag{10}$$

where

$$B = H^2 \tag{11}$$

$$D = 8\rho'/9\alpha \tag{12}$$

$$\rho' = r_0 - \omega'^2 + 3\alpha H_0^2 \tag{13}$$

$$u = \frac{16}{27\alpha^2}\left(3k_0^2\omega'^2 - \rho'^2\right) \tag{14}$$

$$v = -\frac{16}{729\alpha^3}\left[8\rho'\left(\rho'^2 + 9k_0^2\omega'^2\right) + 81\alpha P'^2\right] \tag{15}$$

Eqn.(10) is standard catastrophic manifold equation of cusp, where B+D is known as state variable, u and v are named as control variables. Because the state variable is made up of B and D variables, so Eqn.(10) is the catastrophic model of double cusp which is made up of two catastrophic model of cusp in fact. The equilibrium surface and the control variable surface of the catastrophic model are shown in Fig.4.

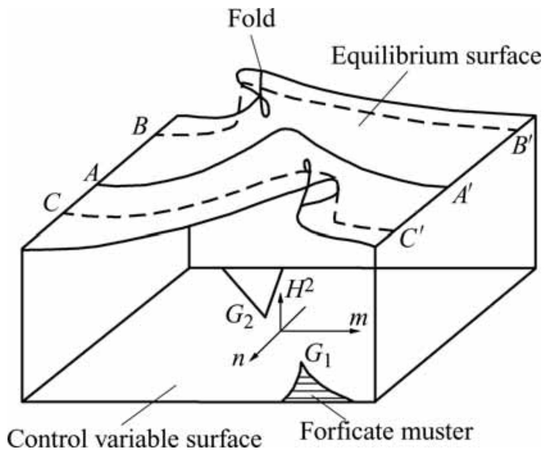

Fig.4 Catastrophic model of double cusp

According to Eqn.(13), it can be known that ρ' expresses the relationship between the frequency of blasting vibration and the natural frequency of the beam when the deadweight of the beam is considered [15]. Since the deadweight of the beam is considered, the natural frequency of the beam turns to ω' from ω_0' which is the value of natural frequency of the beam when the deadweight of the beam is not considered.

$$\omega_0 = \sqrt{\omega_0'^2 + 3\alpha H_0^2} \tag{16}$$

Let u=0, v=0, then the two cusps , $G_1(m_1, n_1)$ and $G_2(m_2, n_2)$, of the catastrophic model of double cusp in Fig.4 can be determined easily.

Let

$$m = \omega'^2 - \omega_0^2$$

then

$$m_{1,2} = k_0\left(\frac{3}{2}k_0 \pm \sqrt{\frac{9}{4}k_0^2 + 3\omega_0^2}\right) \tag{17}$$

$$n_{1,2} = P_{1,2}'^2 = \frac{32m_{1,2}^3}{81a} \tag{18}$$

4 Critical safety thickness

4.1 Instability conditions

According to the catastrophic model of double cusp established above, the condition of destabilization of underground chambers is induced by blasting vibration. It is known from catastrophe theory[7] that the structure will change suddenly under minute external disturbance when it is at critical state (locating at the forficate cluster), so the forficate cluster equation of catastrophic model is the sufficient condition of catastrophe.

To get the derivate of Eqn.(10), then

$$3(B+N)^2+u=0 \tag{19}$$

Based on the simultaneous Eqns.(10) and (19), bifurcation cluster equation can be obtained,

$$4u^3+27v^2=0 \tag{20}$$

Substituting Eqn.(14) and Eqn.(15) into Eqn.(20) gives

$$4\left[16\left(3k_0^2\omega'^2-\rho'^2\right)/27\alpha^2\right]^3+$$
$$27\left\{16\left[8\rho'\left(\rho'^2+9k_0^2\omega'^2\right)+81\alpha P'^2\right]/729\alpha^3\right\}^2=0 \tag{21}$$

Eqn.(21) is a sufficient condition that blasting vibration induces the destabilization of underground chambers. Not only external disturbance factors, such as the influence of deadweight of the beam and blasting vibration signal, but also internal property of structure, such as geometrical dimension and material behavior, is included in Eqn.(21). Therefore, whether blasting vibration can induce the destabilization of underground chambers or not is not only dependent on the amplitude and frequency of blasting vibration, but also related to deadweight of the beam and intrinsic property of structure under the circumstance of deadweight of the beam.

4.2 Critical safety thicknesses

According to solid mechanics theory, the bending value of the beam ω in Fig.2 is inversely proportional to the thickness of the beam h. When the other conditions are certain value, the bigger the h, the less the ω, and the safer the roof (floor) of underground chambers, and the more difficult the destabilization and damage to happen. By contrast, the destabilization and damage of underground chambers are easier to happen. So there is a critical value h_0 of the thickness of the beam. The destabilization and damage of the roof (floor) of underground chambers will take place when the thickness of the beam h is larger than h_0. Otherwise, the roof (floor) of underground chambers is safe.

Further developing Eqn.(21) and collating gives

$$81^2P'^4\alpha^2+162rP'^2+r^2+s=0 \tag{22}$$

where

$$r=8\rho'\left(\rho'^2+9k_0^2\omega'^2\right) \tag{23}$$

$$s=-64\left(3k_0^2\omega'^2-\rho'^2\right)^3>0 \tag{24}$$

Solving Eqn.(22), and neglecting the less value, then

$$\alpha=\frac{r+\sqrt{s}}{81P'^2} \tag{25}$$

Substituting $\alpha=\pi^4EA/(4\rho L^4)$ into Eqn.(25) and collating gives

$$A=\frac{4\rho L^4\left(r+\sqrt{s}\right)}{81\pi^4EP'^2} \tag{26}$$

Under the circumstance of unit width, $A=h$. Substituting it into Eqn.(26) gives

$$h=\frac{4\rho L^4(r+\sqrt{s})}{81\pi^4EP'^2} \tag{27}$$

Now, the thickness of the beam h is the critical safety thickness h_0, which is the critical safety thickness of the roof (floor) of underground chambers under blasting vibration dynamic load under the circumstance of deadweight of the beam.

5 Influence of intension and frequency on h_0

5.1 Influence of intension

According to situ test and above derivation, it is shown that the influence of the amplitude of blasting vibration P' on the critical thickness h_0 is relatively serious. The quantitative relation between the amplitude P' and h_0 will be discussed in the following. When the deadweight of the beam is not considered, it is assumed that ω_0=251.2 rad/s, ω'=62.8 rad/s, ρ=3.0 t/m^3, k_0=0.1, L=20 m, E=6 GPa. According to Eqn.(13), Eqn.(23), Eqn.(24), Eqn.(27), we obtain

$$h_0=\frac{135\times10^6}{P'^2} \tag{28}$$

Because the observed physical quantity of situ blasting vibration test is generally vibration velocity of particle, the relation between h_0 and vibration velocity of particle v' should be determined. According to Ref.[17], the dynamic stress at any point in rock mass is

$$\sigma = \rho c' \times v' \tag{29}$$

where c' is the velocity of longitudinal wave in rock mass; $\rho c'$ is wave impedance of rock mass; v' is the vibration velocity of particle (that is velocity response of particle).

And

$$P' = \sigma A = \sigma h_0 \tag{30}$$

Substituting Eqn.(29) and Eqn.(30) into Eqn.(28) gives

$$h_0 = \sqrt[3]{\frac{135\times 10^6}{(pc')^2 v'^2}} \tag{31}$$

The wave impedance of rock mass $\rho c'$ is assumed to be 1.5×10^3 kg/(m^2·s), then the above equation becomes

$$h_0 = \sqrt[3]{\frac{6}{v'^2}} \tag{32}$$

The type curve of h_0—v' is shown in Fig.5. The unit of frequency ω' is rad/s. According to SADOWSKY expressions, the relation between the vibration velocity of particle v' and blasting dose Q can be determined. Then, according to Eq.(32), the relation between h_0 and blasting dose Q can be confirmed further.

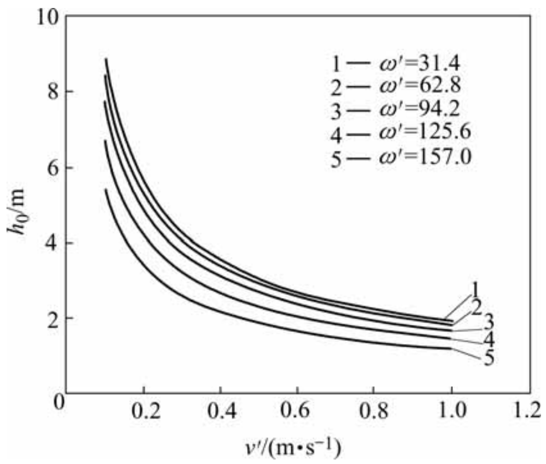

Fig.5 Relation curve of h_0—v'

Some points can be understood from Fig.5:

1) With accretion of the vibration velocity of particle v', the critical safety thickness of the roof (floor) of underground chambers h_0 decreases gradually. That is to say, the larger the vibration velocity of particle v', the easier the destabilization of underground chambers under blasting vibration.

2) When the vibration velocity of particle v' is relatively low（lower than 0.2 m/s）, the critical safety thickness h_0 is larger. And it is difficult that the destabilization of underground chambers takes place now, which is proved by a lot of situ test and investigation. However, when the vibration velocity of particle v' is relatively high (＞0.5 m/s), the degree of decreasing of h_0 becomes little and it tends to be stable gradually with the vibration velocity of particle v' increasing. It is illuminated that the sensitivity of h_0 decreases with the increasing of blast scale.

3) When the vibration velocity of particle v' is certain, the critical safety thickness of the roof (floor) of underground chambers h_0 decreases gradually with accretion of the frequency of blasting vibration ω'. It is shown that the frequency of blasting vibration ω' will influence the stability of underground chambers, which is proved by situ test and the latest research results. And the h_0—v' curve becomes smooth gradually, and the sensitivity of h_0 decreases with the increasing of frequency of blasting vibration ω'.

5.2 Influence of frequency

The quantitative relation between the amplitude ω' and h_0 can be confirmed according to Eqns.(13), (25), (26), and (29), too. The type h_0—ω' curve is shown in Fig.6. It can be found from Fig.6 that with accretion of the frequency of blasting vibration ω', the critical safety thickness of the roof (floor) of underground chambers h_0 decreases gradually. It is because that ω' is close to the natural frequency of the beam ω_0 (ω_0=251.2 rad/s). If the frequency of blasting vibration ω' increases to the natural frequency of the beam ω_0, then resonance effect will happen. And the critical safety thickness of the roof (floor) in underground chambers h_0 will become zero, which denotes that the roof (floor) of underground chambers has been damaged and broken, and its loading capacity has lost.

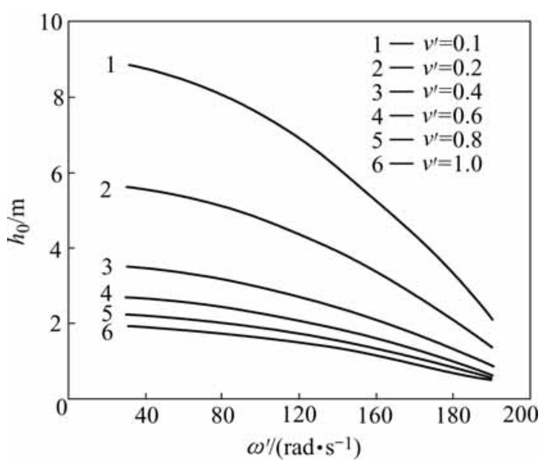

Fig.6 Relation curve of h_0—ω'

6 Conclusions

1) The destabilization of underground chambers induced by blasting vibration is a discontinuous,

nonlinear mechanical process. In virtue of catastrophe theory, the mechanism of destabilization of underground chambers induced by blasting vibration is analyzed. It provides the foundation for revealing the failure mechanism of underground chambers under blasting vibration dynamic load.

2) Within some range of the intension and frequency of blasting vibration, the dynamic response of the roof (floor) of underground chambers will cross over the critical equilibrium position, and force the amplitude (displacement) of structure to jump.

3) The process of destabilization of underground chambers induced by blasting vibration is irreversible. The response of underground chambers to blasting vibration load is hysteretic, that is to say, the change path of dynamic load frequency brings important influence to vibration characteristics of structures.

4) Whether blasting vibration can induce the destabilization of underground chambers or not is not only dependent on the intension and frequency of blasting vibration, but also related to deadweight of the beam and internal property of structure under the circumstance of deadweight of the beam.

5) With accretion of amplitude and frequency of blasting vibration, the critical safety thickness of the roof (floor) of underground chambers decreases gradually, and the possibility of underground chambers destabilization increases.

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