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Constitutive equation of annealed copper with high conductivity for deformation at high strain rates①

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Abstract: After the discussion of a lot of constitutive equations, Zerilli-Armstrong constitutive equation (Z-A equation) was found to be a quasi-static equation. Based on this Z-A equation, a constitutive relations equation was constructed for dynamical calculation of fcc matels such as OFHC based on the thermal viscoplastic relations, where thermal dynamical parameters are related to the evolution of the microstructure of the deforming metals, and the variation of the density of the mobile dislocation was also considered. Data from the deformation of annealed copper were used to fit the parameters in the equation. The predicting results by using the constitutive equation are in good agreement with the experimental data.

Key words: fcc metals; dynamical calculation; constitutive relation

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1 INTRODUCTION

In the deformation process of materials, flow stress depends on several factors such as strain-rate, strain and temperature. Usually flow stress increases with the increase in strain and the decrease in temperature. However, stain-rate has complicated effects on flow stress. Flow stress may increase with increasing in strain-rate because some dislocations in the deforming material don't have enough time to move.

However, if the strain-rate is too high, the deformation heat doesn't have enough time to transfer to the environment. As a result, the deformation temperature will be raised and the material will be softened. The value of the flow stress depends on the balance of the above two tendencies. Additionally, the flow stress strongly depends on the microstructures of the deforming material. Therefore, microstructure parameters and thermal-dynamics parameters such as strain, strain-rate and temperature should be considered to construct an ideal constitutive equation of materials. A commonly used type of constitutive equation is as[1]

$$\left.\begin{aligned}\dot{\varepsilon} &= f(\sigma, T, \tau_1, \tau_2, \ldots, \tau_n)\\ \frac{\mathrm{d}\tau_i}{\mathrm{d}t} &= \dot{\varepsilon}\frac{\mathrm{d}\tau_i}{\mathrm{d}\varepsilon} = g(\tau, T, \tau_1, \tau_2, \ldots, \tau_n)\end{aligned}\right\} \quad (1)$$

where $\dot{\varepsilon}$ is strain-rate, σ is stress, T is temperature and τ_i is microstructural parameter.

2 DERIVATION AND ANALYSIS

Many researchers have devoted to construction of constitutive equations such as Klepaczko-Chiem equation[2], Mecking-kocks equation[3], Johnson-Cook equation[4], Zerilli-Armstrong equation[5] and Nemat-Nasser model[6]. However, all these models have their limitations. Klepaczko-Chiem equation is not suitable for strain-rate higher than $10^3\ s^{-1}$ and Nemat-Nasser model can just be used for relatively low stain-rate because it is based on the assumption that the flow stress monotonously increases with strain-rate. It is well-known that deformation heat at very high strain rate will probably soften the deforming material and lower the value of flow stress. To explain the deforming behavior of materials at a strain-rate higher than $10^3\ s^{-1}$, another model was proposed[7]:

$$\sigma = \sigma_b + \beta\dot{\varepsilon} \quad (2)$$

where σ_b and β are material constants. This model can express the relations among stress, strain and strain-rate, but there are some shortcomings in this equation. If the equation is used to express the exponent of the strain rate sensitivity of materials, Eqn. (2) can be written as[7]

$$\frac{\partial \ln\sigma}{\partial \ln\dot{\varepsilon}} = \frac{\beta\dot{\varepsilon}}{\sigma_b + \beta\dot{\varepsilon}} \quad (3)$$

It should be noted that the predicted value by Eqn. (3) is 10－100 times larger than the experimen-

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tal one for materials with given microstructure[7]. Some models have been constructed in China, but most of these models are non-physically-based or can just be applied to low strain-rate[8-13].

To construct a constitutive equation for dynamical calculation at high strain rate, the Zerilli-Armstrong constitutive equation for fcc materials will be analyzed at first.

According to Armstrong, there is an equation as[5]

$$\sigma_{th} = B\exp(-\beta T) \tag{4}$$

where σ_{th} is uniaxial sress, while

$$B = mG_0/A_0 b \tag{5}$$

and

$$\beta = (1/T)\ln(A/A_0) - (1/T)\times\ln[1+(kT/G_0)\ln(\dot{\varepsilon}/\dot{\varepsilon}_0)] \tag{6}$$

where m is tensor orientation factor. T is absolute temperature, G_0 is reference Gibbs energy at $T = 0$, k is Boltzmann's constant, A is the area of activation, A_0 is the dislocation activation area at $T = 0$, and b is the Burgers vector.

$$\dot{\varepsilon} = (m'/m)bNv_0 \tag{7}$$

where m' is also tensor or irientation factor

It is proved[14] that

$$\beta = \beta_0 - \beta_1\ln(\dot{\varepsilon}/\dot{\varepsilon}_0) \tag{8}$$

Eqns. (4)~(8) are considered together and let $\ln(1+x)\approx x$, then

$$\sigma_{th} = C_1\exp(-C_3T + C_4T\ln\dot{\varepsilon}) \tag{9}$$

Compare Eqn. (9) with Eqns. (4) and (5), then

$$C_1 = mG_0/(A_0 b) \tag{10}$$

As to fcc materials, if the thermal activated area of dislocations is used to evaluate the distance between the spots of the dislocation interactions, then

$$A \approx db/2 \tag{11}$$

where d is the distance between the interactions of dislocations, and it has relations with N, the density of dislocations, as

$$N \sim 1/d^2 \tag{12}$$

Tanner et al[15] reported that the relations between flow stress and the density of dislocations were as

$$\sigma_o \approx \sigma'_0(b/d) \approx \sigma''_0\varepsilon^{1/2} \tag{13}$$

where σ'_0, σ''_0 are material constants. A_0 can be calculated by Eqns. (11)~(13), as

$$A_0 = A'_0\varepsilon^{1/2} \tag{14}$$

where A'_0 is a constant just concerning with σ'_0, σ''_0. From Eqns. (14), (9) and (10), there are

$$\sigma_{th} = C_2\varepsilon^{1/2}\exp(-C_3T + C_4T\ln\dot{\varepsilon}) \tag{15}$$

The model is derived from the mechanism of the dislocation thermal activation. If the effects of solution atoms on the flow stress are $\Delta\sigma_G$ and the effects of the grain sizes on the flow stress are:

$$\Delta\sigma_l = kl^{-1/2} \tag{16}$$

From Eqns. (15) and (16), there are relations as

$$\sigma = \Delta\sigma_G + kl^{-1/2} + C_2\varepsilon^{1/2}\exp(-C_3T + C_4T\ln\dot{\varepsilon}) \tag{17}$$

3 COEFFICIENT ANALYSIS AND CONSTRUCTION OF CONSTITUTIVE EQUATION

In Eqn. (17), $\Delta\sigma_G$, $kl^{-1/2}$ show the effects of solution atoms and the sizes of grains which can be regarded as constant in the whole deforming process of materials. According Eqns. (9), (10), (14) and (15), the coefficient C_2 can be calculated as

$$C_2 = \frac{mG_0}{bA'_0} \tag{18}$$

It is known in the above equation that C_2 is a constant just concerning with Burgers vector, the reference Gibbs energy and tensor orientation factor and it doesn't vary in the whole deforming process. C_4 can be calculated from Eqns. (6)~(9) as

$$C_4 = \frac{k}{G_0} \tag{19}$$

And C_4 is just concerning with reference Gibbs energy and Boltzmann's constant.

With the similar derivation of C_4, C_3 can be calculated as

$$C_3 = (1/T)\ln(A/A_0) + \frac{k}{G_0}bv_0\ln(m'/m) + \frac{k}{G_0}\ln N \tag{20}$$

Experiment proves that the first term of Eqn. (20) is constant[14]. The second term is made up of the reference dislocation velocity, the orientation factor, the Burgers vector and the reference Gibbs energy which have nothing to do with the deforming process. N is considered as the total density of the dislocations and as a constant in the equation of Armstrong-Zerrilli. Actually, N should be the density of the mobile dislocations[1]. It is well known that the total density of dislocation in materials includes two parts. One is the mobile dislocations, the other is the fixed ones. The deformation of material is the results of the movement of mobile dislocations and strain-rate directly shows the speed of the movement of the mobile dislocation. The density of the mobile dislocations varies with strain. In the process of deformation, the increasing of the total dislocation density will make it easier for the plastic flowing of the material and soften the material, but at the same time, the possibilities of dislocation interactions will increase, which will lower the percentage of the dislocation density in the total and make it difficult for the plastic flowing of the deforming material. The evolution of the total dislocation density can't explain the deformation behavior of materials directly. Therefore N should be considered as the mobile dislocation density which can not be considered as constant in the deforming pro-

cess. However, the evolution of the material microstructure is very complicated. A mean ideal is used to describe this evolution. A simple empirical formula is as[5]

$$\rho = (\rho_0 + M\varepsilon)\exp(-\varphi\varepsilon) \quad (21)$$

where ρ is the density of mobile dislocation, ρ_0 is the original dislocation density, M is a material constant concerning with the increasing rate of dislocation density, φ is a decreasing coefficient of the mobile dislocation density with increasing of strain, $\exp(-\varphi\varepsilon)$ is the percentage of the mobile dislocation density in the total. Eqn. (21) shows the evolution of the material microstructure based on the accumulation and recovery of dislocations. In Eqn. (20), let

$$C'_3 = (1/T)\ln(A/A') + \frac{k}{G_0} bv_0 \ln(m'/m) \quad (22)$$

$$C_0 = \Delta\sigma_G + kl^{-1/2} \quad (23)$$

From Eqns. (18)–(23) and Eqn. (17), the flow-stress can be calculated as

$$\sigma = C_0 + C_2\varepsilon^{1/2}\exp(-C'_3 T + C_4\varphi\varepsilon + C_4 T\ln\frac{\dot{\varepsilon}}{\rho_0 + M\varepsilon}) \quad (24)$$

where the parameters C_0, C_2, C'_3, C_4, ρ_0, M, φ are decided by experiments. Eqn. (24) describes the relations between the macro parameters of deformation and the evolution of the microstructure. In the above equation, if the strain equals zero, then the flow stress equals C_0. Therefore C_0 is a parameter concerning with initiate microstructure of the material.

4 ASSESSMENT OF EQUATION

Experimental data from the deformation of fully annealed copper are used to fit Eqn. (24)[4]. Because the copper has been fully annealed, then let $\rho_0 = 0$. According to Armstrong-Zerrilli[4], the parameter C_0 is 65 MPa, and then Eqn. (24) can be written as

$$\ln(\sigma - C_0) = C_2 + C_6 T + C_4 T\ln(\dot{\varepsilon}/\varepsilon) + 0.5\ln\varepsilon \quad (25)$$

According to the least square method, a software of Lingo2 has been used, the parameters in Eqn. (25) can be worked out as: $C'_2 = -4.8$, $C_4 = 0.001\,64$, $C_6 = 0.017\,6$.

The same method and data are used to fit the classic Zerrilli-Armstrong equation and B is used to take the place of $\Delta\sigma_G + kl^{-1/2}$, then the parameters of the Z-A equation can be calculated as: $B = 60$ MPa, $C_2 = 380$ MPa, $C_3 = 0.003\,1\ \mathrm{K}^{-1}$, $C_4 = 0.000\,25\ \mathrm{K}^{-1}$. Fig. 1 shows the predictions of the two equations and the experimental data.

The fitness of the equations can be calculated as

$$\delta = \frac{\sum_{i=1}^{n} \frac{|\sigma_{calc}(\varepsilon_i) - \sigma_{exp}(\varepsilon_i)|}{\sigma_{exp}(\varepsilon_i)}}{n} \quad (26)$$

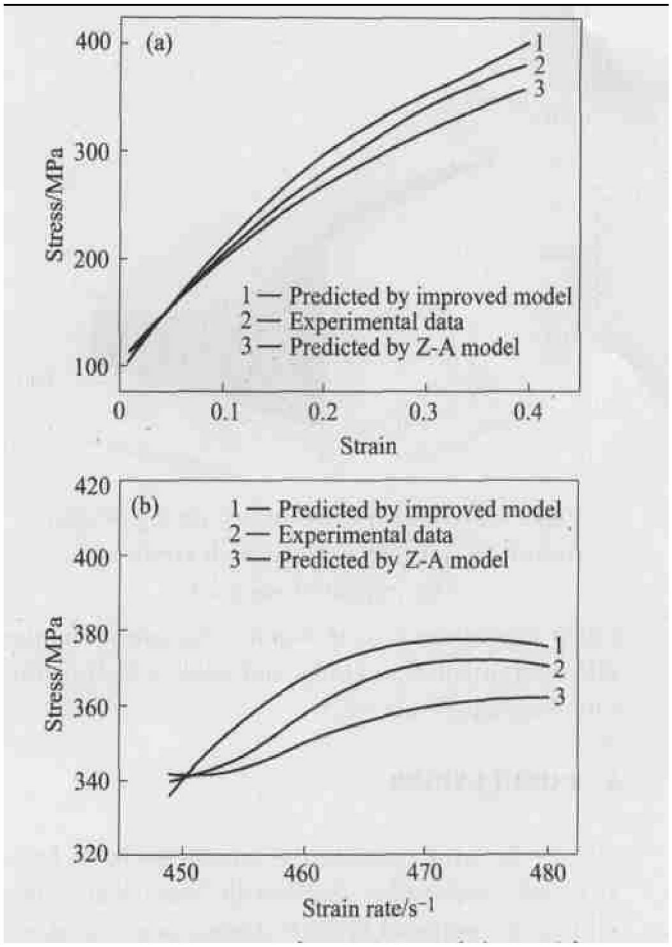

Fig. 1 Comparison of experimental data with predictions of Zerrilli-Armstrong equation and improved equation at 294 K

(a) —Curves of stress—strain at strain rate of 451 s^{-1}; (b) —Curves of stress—strain rate at strain of 0.3

where $\sigma_{calc}(\varepsilon_i)$ is the model predicting stress at a strain of ε_i; $\sigma_{exp}(\varepsilon_i)$ is the experimental data at a strain of ε_i; n is the data number.

It is calculated that the fitness of the classic Z-A equation is 0.11 and the fitness of the improved equation is 0.04, which means a higher accuracy of the improved equation. The main reason is that the classic Z-A equation considered some structural factors as constant. Actually, Z-A equation is a quasi-static equation.

Fig. 2 shows the prediction of the relations between stress, strain-rate and temperature at a strain of 0.25 by the improved equation.

As can be seen in Fig. 2, the flow stress drops down very quickly with the rising deforming temperature. Moreover, the flow stress increases slightly with increasing strain-rate within the strain-rate lower than 600 s^{-1}. This is in correspondence with the experimental data because the movement of dislocations in the material at a relatively low strain-rate is mainly dependent on the diffusion of solution atoms

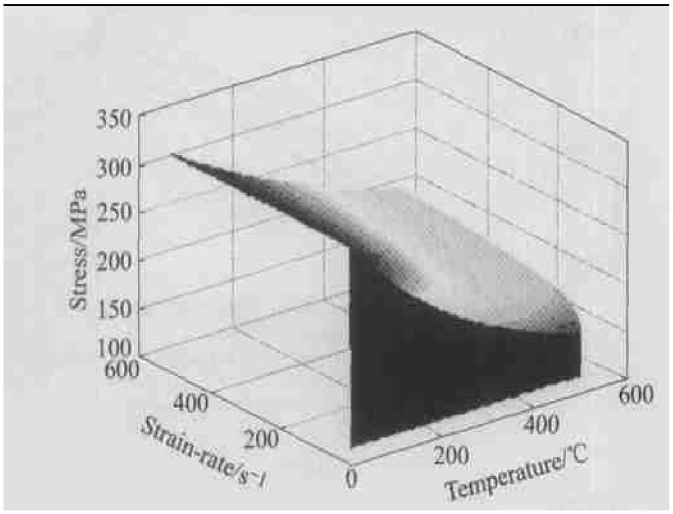

Fig. 2 Prediction of relations among stress, strain-rate and temperature with strain of 0.25 by improved equation

and voids which firmly rely on temperature.

5 CONCLUSIONS

1) Several constitutive equations have been analyzed. Especially, the Zerrilli-Armstrong equation for fcc material at high strain—rate was studied. It is found that the Zerrilli-Armstrong equation essentially is a quasi-static equation.

2) A simple and practicable constitutive equation is constructed by introducing an equation concerning with the microstructure evolution of the deforming material based on the quasi-static Z-A equation. Data from the deformation of the fully annealed copper have been used to make assessment of the improved equation. The equation is found to have a high accuracy of predictions.

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