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Solution for slab forging with bulge between two parallel platens by strain rate vector inner-product integration and series expansion①

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Abstract: A new linear integration was developed. First, effective strain rate for slab forging with bulge was expressed in terms of two-dimensional strain rate vector, and its inner-product was integrated term by term. Second, a summation process of term by term integrated results and a formula of the bulging were introduced, and an analytical solution of stress effective factor was obtained. It is proved that the expression of power by the above linear integration is the same as that of traditional immediate integration. Also, the solution was simplified by series expansion and compared by slab forging test with the others. It turns out that the calculated result of total forging pressure is basically in agreement with measured value in the actual press test.

Key words: slab forging; bulge; strain rate vector; inner-product; linear integration
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1 INTRODUCTION

Slab forging with bulge between two parallel platens is shown in Fig. 1. Friction over the surface decreases the velocity v_x at the surface, and causes the center $y=0$ to move faster with a resultant bulge as shown in Fig. 1(a). Therefore, this velocity gradient from the surface to the interior introduces a shear strain rate $\dot{\varepsilon}_{xy}$.

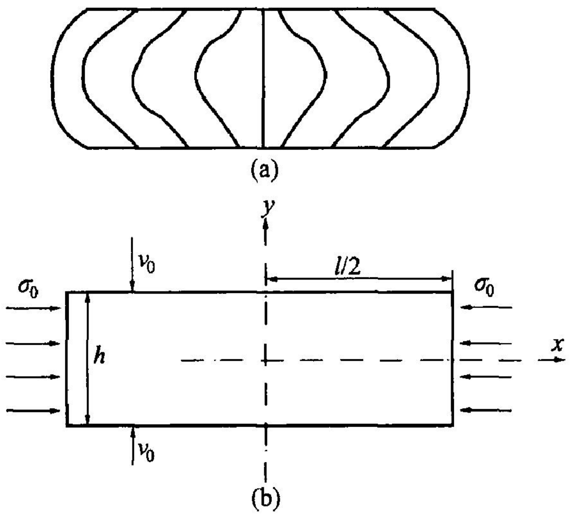

Fig. 1 Slab forging with bulge between two parallel platens

It is the bulge that make the forging more difficult to be analytically solved[1]. However, numerical methods[2], including FEM[3] and UBEM[4] as well as numerical simulation[5], have gradually come to their stage of maturity in recent years. In the present paper, the aim is to seek analytical solution by so-called inner-product of strain rate vector[6, 7], and then compare it with traditional immediate integration[8-10].

2 STRAIN RATE TENSOR FIELD

Assuming that v_x varies exponentially with the y coordinate as shown in Fig. 1(b), the velocity field becomes

$$\left.\begin{aligned} v_x &= \frac{b}{1-e^{-b}}v_0\frac{2x}{h}e^{-2by/h} \\ v_y &= \frac{1}{1-e^{-b}}v_0(e^{-2by/h}-1) \\ v_z &= 0 \end{aligned}\right\} \quad (1)$$

The strain rate components are found when Cauchy equation is applied to the velocity field of Eqn. (1):

$$\left.\begin{aligned} \dot{\varepsilon}_x &= \frac{2bv_0}{(1-e^{-b})h}e^{-2by/h} \\ &= -\dot{\varepsilon}_y \\ \dot{\varepsilon}_{xy} &= \frac{1}{2}\left(\frac{\partial v_x}{\partial y}+\frac{\partial v_y}{\partial x}\right) \\ &= \frac{-2b^2v_0x}{(1-e^{-b})h^2}e^{-2by/h} \\ \dot{\varepsilon}_{zx} &= \dot{\varepsilon}_{yz} = \dot{\varepsilon}_z = 0 \end{aligned}\right\} \quad (2)$$

The detail deduction process of the above

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equations could be found in Refs. [8, 9]. Those strain rate components satisfy

$$\frac{\dot{\varepsilon}_{xy}}{\dot{\varepsilon}_x}=\frac{\dfrac{-2b^2v_0x}{(1-e^{-b})h^2}e^{-2by/h}}{\dfrac{2bv_0}{(1-e^{-b})h}e^{-2by/h}}=-b\frac{x}{h} \tag{3}$$

It must be pointed out that "−" should be added before v_y in Eqn. (13. 13c) of Ref. [8]. Then it would be the second formula[11] in Eqn. (1).

3 POWER OF DEFORMATION

3.1 Inner-product of strain rate vector for plastic power

The effective strain rate in the integrand of plastic power is expressed in inner-product of strain vector, the following is obtained:

$$\begin{aligned}\dot{W}_i&=\int_V\sigma_e\dot{\varepsilon}_e\mathrm{d}V\\&=2k\int_V\sqrt{\dot{\varepsilon}_x^2+\dot{\varepsilon}_{xy}^2}\,\mathrm{d}V\\&=8k\int_0^{l/2}\int_0^{h/2}|\dot{\varepsilon}||\dot{\varepsilon}^0|\cos(\dot{\varepsilon},\dot{\varepsilon}^0)\,\mathrm{d}x\,\mathrm{d}y\\&=8k\int_0^{l/2}\int_0^{h/2}\dot{\varepsilon}\dot{\varepsilon}^0\,\mathrm{d}x\,\mathrm{d}y\end{aligned}$$

where $\dot{\varepsilon}=\dot{\varepsilon}_x\boldsymbol{e}_1+\dot{\varepsilon}_{xy}\boldsymbol{e}_2$ and $\dot{\varepsilon}^0=l_1\boldsymbol{e}_1+l_2\boldsymbol{e}_2$.

$$\begin{aligned}\dot{W}_i&=8k\int_0^{l/2}\int_0^{h/2}(\dot{\varepsilon}_xl_1+\dot{\varepsilon}_{xy}l_2)\,\mathrm{d}x\,\mathrm{d}y\\&=8k\int_0^{l/2}\int_0^{h/2}\left[\frac{\dot{\varepsilon}_x\dot{\varepsilon}_x}{\sqrt{\dot{\varepsilon}_x^2+\dot{\varepsilon}_{xy}^2}}+\frac{\dot{\varepsilon}_{xy}\dot{\varepsilon}_{xy}}{\sqrt{\dot{\varepsilon}_x^2+\dot{\varepsilon}_{xy}^2}}\right]\mathrm{d}x\,\mathrm{d}y\\&=8k[I_1+I_2]\end{aligned} \tag{4}$$

For Eqn. (4), term by term integration[6, 7] was used. And notice Eqn. (3), it follows that

$$\begin{aligned}I_1&=\int_0^{l/2}\int_0^{h/2}\frac{\dot{\varepsilon}_x\,\mathrm{d}x\,\mathrm{d}y}{\sqrt{1+\left[\dfrac{\dot{\varepsilon}_{xy}}{\dot{\varepsilon}_x}\right]^2}}\\&=\frac{2bv_0}{(1-e^{-b})h}\int_0^{l/2}\int_0^{h/2}\frac{e^{-2by/h}\,\mathrm{d}x\,\mathrm{d}y}{\sqrt{1+\left[-b\dfrac{x}{h}\right]^2}}\\&=v_0\frac{h}{b}\ln\left[\frac{bl}{2h}+\sqrt{1+\left[\frac{bl}{2h}\right]^2}\right]\end{aligned} \tag{5a}$$

$$\begin{aligned}I_2&=\int_0^{l/2}\int_0^{h/2}\frac{\dot{\varepsilon}_{xy}\,\mathrm{d}x\,\mathrm{d}y}{\sqrt{1+\left[\dfrac{\dot{\varepsilon}_{xy}}{\dot{\varepsilon}_x}\right]^{-2}}}\\&=-v_0\left\{\frac{l}{4}\sqrt{1+\frac{b^2l^2}{4h^2}}-\frac{h}{2b}\ln\left[\frac{bl}{2h}+\sqrt{1+\frac{b^2l^2}{4h^2}}\right]\right\}\end{aligned} \tag{5b}$$

Substitution of Eqns. (5a) and 5(b) into Eqn. (4) leads to

$$\begin{aligned}\dot{W}_i&=8k[I_1+I_2]\\&=4kv_0\left\{\frac{h}{b}\mathrm{Arsh}\frac{bl}{2h}+\frac{l}{2}\sqrt{1+\left[\frac{bl}{2h}\right]^2}\right\}\end{aligned} \tag{5c}$$

Eqn. (5c) is analytical solution for internal power of deformation, which is integrated term by term and summed according to strain vector inner-product. It should be noticed that there is an arc-hyperbolic sine in Eqn. (5c) and it's the same as that of Avitzur's immediate integration[8, 9].

3.2 Friction losses and power to overcome resistance

Here constant friction ($\tau_f=mk$) is assumed and the velocity discontinuity by Eqn. (1) is

$$\begin{aligned}\Delta\bar{v}_f&=v_x\big|_{y=h/2}\\&=\frac{2}{1-e^{-b}}v_0\frac{2x}{h}e^{-b}\end{aligned}$$

Friction losses become

$$\begin{aligned}\dot{W}_f&=\int_{F_f}\tau_f|\Delta v_f|\,\mathrm{d}F\\&=\frac{4mk\cdot 2bv_0}{(1-e^{-b})h}e^{-b}\int_0^{l/2}x\,\mathrm{d}x\\&=mk\frac{be^{-b}v_0}{(1-e^{-b})}\frac{l^2}{h}\end{aligned} \tag{6}$$

If external stress σ_0 is applied at the free surfaces $x=\pm\dfrac{l}{2}$, then the power to overcome the resistance of this pressure is

$$\begin{aligned}\dot{W}_b&=4\times\frac{h}{2}v_x\sigma_0\\&=4\times\frac{h}{2}\frac{v_0l}{h}\sigma_0\\&=2lv_0\sigma_0\end{aligned} \tag{7}$$

4 TOTAL POWER AND MINIMIZATION

4.1 Total power and series expansion

Substitution of Eqns. (5(c)), (6) and (7) into $J^*=\dot{w}_i+\dot{w}_f+\dot{w}_b$ leads to

$$J^*=4kv_0\left\{\frac{h}{b}\mathrm{Arsh}\frac{bl}{2h}+\frac{l}{2}\sqrt{1+\left[\frac{bl}{2h}\right]^2}\right\}+mk\frac{be^{-b}v_0}{(1-e^{-b})}\frac{l^2}{h}+2lv_0\sigma_0 \tag{8}$$

When

$$2\bar{p}lv_0=J^*$$

$$\frac{\bar{p}}{2k}=\frac{h}{bl}\mathrm{Arsh}\frac{bl}{2h}+\frac{1}{2}\sqrt{1+\left[\frac{bl}{2h}\right]^2}+\frac{me^{-b}}{(1-e^{-b})}\frac{bl}{4h}+\frac{\sigma_0}{2k} \tag{9}$$

If no external pressure exists, then $\sigma_0=0$.

The upper-bound analytic solution of slab forging is obtained by Eqn. (9).

Expanding Eqn. (9) into series:

$$\mathrm{Arsh}\left[\frac{bl}{2h}\right]=\left[\frac{bl}{2h}\right]-\frac{1}{2\times 3}\left[\frac{bl}{2h}\right]^3+\cdots,$$

$$\left|\frac{bl}{2h}\right|<1$$

$$\left[1+\frac{bl}{2h}\right]^{1/2}=1+\frac{1}{2}\left[\frac{bl}{2h}\right]-\cdots,$$

$$\left|\frac{bl}{2h}\right| \leqslant 1$$

$$e^{-b} = 1 + \frac{-b}{1!} + \frac{(-b)^2}{2!} + \cdots,$$

$$|-b| < \infty$$

These expansions hold if $\frac{bl}{2h} < 1$.

Omitting terms to the second power and higher:

$$\frac{\bar{p}}{2k} = \frac{h}{bl}\left[\frac{bl}{2h} - \frac{1}{6} \times \frac{b^3 l^3}{8h^3}\right] + \frac{1}{2}\left[1 + \frac{1}{2}\left[\frac{bl}{2h}\right]^2\right] + \frac{m(1-b)}{1-(1-b)}\frac{bl}{4h} + \frac{\sigma_0}{2k}$$

$$\frac{\bar{p}}{2k} = 1 + \frac{b^2 l^2}{24h^2} + \frac{ml}{4h} - \frac{ml}{4h}b + \frac{\sigma_0}{2k} \tag{10}$$

Eqn. (10) is the simplified analytic solution by series expansion.

The arbitrary parameter b is to be so chosen that Eqn. (10) is minimized. Differentiation of Eqn. (10) and equating to zero leads to

$$\frac{\partial(\bar{p}/2k)}{\partial b} = 0 \rightarrow \frac{2bl^2}{24h^2} - \frac{ml}{4h} = 0$$

Therefore,

$$b = m\frac{3h}{l} \tag{11}$$

Substitution of Eqn. (11) into Eqn. (10) leads to the minimum upper-bound solution:

$$n_{\sigma(\min)} = \frac{\bar{p}_{\min}}{2k} = 1 + \frac{ml}{4h} - \frac{3m^2}{8} + \frac{\sigma_0}{2k} \tag{12}$$

This is the minimum upper-bound solution of stress effective factor. $\sigma_0 = 0$ means no external pressure.

Avitzur's approximate solution[8] for the same problem is

$$\frac{\bar{p}}{2k} = 1 + \frac{m}{4}\frac{l}{h} - \frac{3}{2}\frac{\left[\frac{m}{4}\right]^2}{1 + 2\left[\frac{m}{4}\right]\left[\frac{h}{l}\right]} + \frac{\sigma_0}{2k} \tag{13}$$

4.2 Parameter *b* of bulge

By the first equation of Eqn. (1):

$$v_x\big|_{y=0,\ x=l/2} - v_x\big|_{y=h,\ x=l/2} = \frac{b}{1-e^{-b}}v_0\frac{l}{h} - \frac{b}{1-e^{-b}}v_0\frac{l}{h}e^{-b} =$$

$$\frac{bv_0 l}{h} = \Delta v_x\big|_{x=l/2}$$

$$\frac{bl}{h} = \frac{\Delta v_x\big|_{x=l/2}}{v_0}$$

$$b = \frac{h\Delta v_x\big|_{x=l/2}}{lv_0}$$

$$= \frac{h}{lv_0}\left[\frac{l_{1m} - l_0}{t} - \frac{l_{1s} - l_0}{t}\right]$$

$$= \frac{2h}{l\Delta h}\left[\frac{l_1\big|_{y=0} - l_1\big|_{y=h/2}}{2}\right]$$

$$= \frac{2h}{l\Delta h}\Delta l\big|_{x=l/2} \tag{14}$$

where $\Delta l\big|_{x=l/2} = \frac{l_{y=0} - l_{y=h/2}}{2}$. $\Delta l\big|_{x=l/2}$ is the measured length of the forged specimen, which is the difference between the middle and the contact surface along x direction at $x = l/2$.

Substitution of Eqn. (14) into Eqn. (11) leads to the value of m. Then substitution of m into Eqn. (12) leads to the minimum upper bound stress effective factor.

5 VALIDATION BY PRESS TEST

The press test was done with 200 kN universal material testing machine in the State Key Laboratory of Rolling and Automation NEU. A pure lead specimen was compressed from an initial size of $h_0 = 20.27$ mm, $l_0 = 49.73$ mm, $B_0 = 70.12$ mm to a final size of $h_1 = 18.35$ mm, $B_1 = 72.81$ mm, $l_1 = 54.58$ mm. The ram speed is 30 mm/min and the total indicator reading of the machine is 97.2 kN.

5.1 Calculated result

The measured dimensions after compression are $l_m = 55.09$ mm, $l_s = 54.07$ mm. Therefore, $l/h = 54.07/18.35 = 2.9466$, $h/l = 0.3394$, $\Delta h = 1.92$ mm. Since Eqn. (15),

$$b = \frac{2h}{l\Delta h}\Delta l\big|_{x=l/2}$$

$$= \frac{h(55.09 - 54.07)}{l \times 1.92}$$

$$= 0.18029$$

Substitution of these parameters into Eqn. (11) leads to

$m = 0.177$

Substitution of the value of m into Eqn. (12), noticing $\sigma_0 = 0$, leads to

$$\frac{\bar{p}}{2k} = 1 + 0.13 - 0.012 = 1.118$$

From the measured dimension:

$$\varepsilon = \frac{\Delta h}{h_0} = 0.095,$$

$$t = \frac{\Delta h}{\bar{v}} = \frac{2 \times 1.92}{30/60} = 7.68\ \text{s};$$

$$\dot{\varepsilon} = \frac{\varepsilon}{t} = 0.01237\ \text{s}^{-1}$$

According to ε, $\dot{\varepsilon}$, it can be checked out $\sigma_s = 20.06$ MPa[12], then the total compression force $P = 1.118 \times \frac{2}{\sqrt{3}} \times 20.06 \times 53.38 \times 71.85 = 99.322$ kN. Notice that the indicator reading of the machine is 97.2 kN, so the difference between them is

$\Delta = (99.3 - 97.2)/97.2 = 2.16\%$

5.2 Comparison with Avitzur's formula

Substitution of $m=0.177$ into Eqn. (13) and noticing $\sigma_0=0$ leads to

$$\frac{\bar{p}}{2k}=1+0.13-0.00285$$
$$=1.127$$

With the same procedure the total forging force is

$P=100.12$ kN

It is proved that the above result is basically in agreement with that of Eqn. (12).

The author of present paper ever deduced a formula to slab forging with bulge by transmission equation[13] as follows:

$$n_\sigma=\frac{\bar{p}}{2k}=\left[1+\frac{m}{4}\frac{l}{h}\right]\frac{l'}{l} \qquad (15)$$

where l', l are the measured length respectively at the middle $y=0$ and the surface $y=h/2$ of the deformed specimen, and $l'/l\geqslant 1$ is the bulge parameter.

Substitution of $m=0.177$ and other parameter into Eqn. (15) leads to

$$\frac{\bar{p}}{2k}=\left[1+\frac{0.177}{4}\times\frac{54.07}{18.35}\right]\times\frac{55.09}{54.07}$$
$$=1.15$$

Therefore,

$$P=1.15\times\frac{2}{\sqrt{3}}\times 20.06\times 53.38\times 71.85$$
$$=102.17\text{ kN}$$

The difference to the indicator reading is

$$\Delta=(102.17-97.2)/97.2$$
$$=5.1\%$$

It should be pointed out that in above computation the contact area is a product area of average length and wide of the deformed specimen. In addition Slab[14] and slip-line[15] methods, as well as others[16] available can be used to solve this kind of problems.

Let $m=0.1$, 0.2, 0.3, 0.4, 0.5 in turn, and calculate stress effective factor $n_\sigma=\bar{p}/2k$ by Eqns. (12), (13) and (15). These results are compared in Table 1 and Fig. 2.

Table 1 Comparison of stress effective factor n_σ

Eqn	m				
	0.1	0.2	0.3	0.4	0.5
(12)	1.0699	1.1323	1.1872	1.2347	1.2746
(13)	1.0727	1.1437	1.2130	1.2806	1.3467
(15)	1.0939	1.1690	1.2440	1.3191	1.3941

From Table 1 and Fig. 2 a conclusion can be obtained that the value of stress effective factor increases along with the increase of m. The result of Eqn. (15) is higher than the results of Eqns. (12) and (13). The results of Eqns. (12) and (13) are basically consistent.

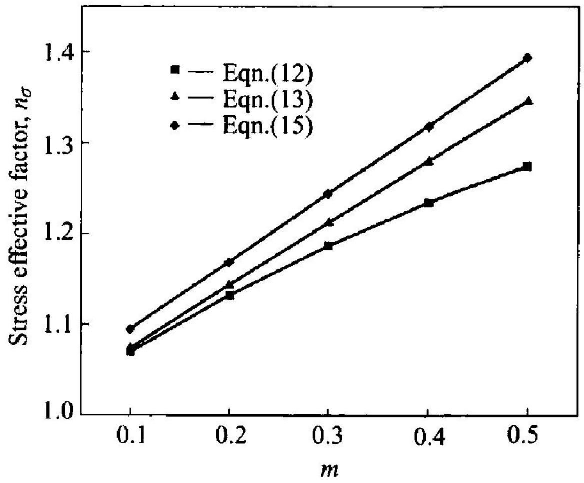

Fig. 2 Comparison of stress effective factor by Eqns. (12), (13) and (15)

6 CONCLUSIONS

1) The effective strain rate for slab forging with bulge can be expressed in terms of two-dimensional strain rate vector, and its inner-product integrated term by term. Then the summation of the integrated results yields an analytical solution of deformation power as shown in Eqn. 5(c). It is arc-hyperbolic sine function, the same as that of traditional immediate integration.

2) By expanding series and power minimization, the simplified stress effective factor Eqn. (12) is obtained. The corresponding bulge parameter b is shown in Eqn. (11).

3) The calculated results of $\bar{p}/2k$ and total forging force by Eqn. (12) is basically in agreement with those by Eqns. (13) and (15). But all of them are a little higher than the value of indicator reading of the machine.

4) It is indicated that stress effective factor increases along with the increase of the value of friction factor m.

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