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Is crack branching under shear loading caused by shear fracture? ——A critical review on maximum circumferential stress theory①

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[Abstract] When a crack is subjected to shear force, crack branching usually occurs. Theoretical study shows that the crack branching under shear loading is caused by tensile stress, but not caused by shear fracture. The co-plane shear fracture could be obtained if compressive stress with given direction is applied to the specimen, subsequently, calculated shear fracture toughness, K_{IIC}, is larger than K_{IC}. A prerequisite of possible occurrence of mode Ⅱ fracture was proposed. The study of shear fracture shows that the maximum circumferential stress theory considered its criterion as a parametric equation of a curve in K_I, K_{II} plane is incorrect; the predicted ratio $K_{IIC}/K_{IC}= 0.866$ is incorrect too.

[Key words] stress intensity factor; fracture toughness; mode of fracture; mode of loading

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1 INTRODUCTION

Classical fracture mechanics always distinguish three basic modes from point of crack surface displacement view: mode Ⅰ (opening mode), mode Ⅱ (sliding mode) and mode Ⅲ (tearing mode). The stress intensity factors and fracture toughness associated with each mode are labeled K_I, K_{II}, K_{III} and K_{IC}, K_{IIC}, K_{IIIC} respectively. Thus there are three modes of fracture. All references relate three modes of fracture with loading modes as shown in Fig. 1[1]. When the study of mode Ⅰ fracture under tensile loading conducted, fruitful results were obtained. Numbers of study works and papers are associated with mode Ⅰ fracture. There are standard test methods to explain how mode Ⅰ fracture test should be conducted and how to calculate mode Ⅰ fracture toughness. The Suggested Method for Determining the Fracture Toughness (K_{IC}) of Rock was firstly published in 1988 by International Society of Rock Mechanics.

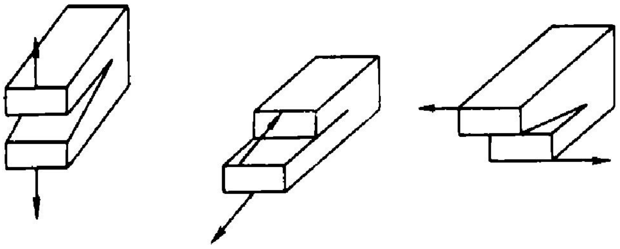

Fig. 1 Three basic modes

However, the study of mode Ⅱ fracture was quite different. Using three different methods (A large plate with a central straight crack subjected to equal and opposite concentrated shear loads applied to the crack surface, a thin-walled tube with a longitudinal crack under torsion and a large plate with a central crack subjected to shear at infinity) Erdogan and Sih[2] conducted shear tests and discovered that the fracture angles measured were around 70° with very small scatter and calculated K_{IIC} was less than K_{IC}, the ratio K_{IIC}/K_{IC} was 0.87. From 1970s to 1990s shear tests were conducted off and on using different methods. Besides above three methods[3, 4], for shear fracture testing the specimens used were anti-symmetric four point bending beam[5~9], compact tension-shear specimen[10], short beam compression specimen[11] and edge cracked Arcan specimen. All test results showed that fracture deviated from original crack plane, the fracture angles varied in range of 60°~ 76°, depending on test methods and shear fracture toughness, K_{IIC}, was less than K_{IC}.

Petit[12] tried to conduct shear test using a large plate with central inclined crack (30°) subjected to uniaxial and biaxial compression. The test results could not change situation of crack branching from original crack plane. Based on theoretical study, Melin[13] considered that a high confining pressure promotes mode Ⅱ growth, but no tests have been conducted.

To do shear tests Izumi[14] used shear box and Tsangarakis[15] designed a special shear device. Their tested results showed co-plane fracture and calculated K_{IIC} being larger than K_{IC}. But they were not self-confident with their results: Tsangarakis[15] could not explain why K_{IIC} obtained by Shah[4] was less than his, when material used was of the same, only test

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methods were different. Their methods were not recognized and have not been noted for mode II fracture tests.

Present fracture theories of mixed loading, for example, the maximum circumferential tensile stress theory[2], the maximum energy release rate theory[16] and the minimum strain energy density theory[17], predicted crack fracture angle under shear loading being $-70.5°$, $-75.6°$, $-70°\sim-80°$ apart from original crack plane and predicted the ratio of K_{IIC}/K_{IC}, equal to 0.866, 0.63[16] and 0.92[17] respectively.

Shear fracture and determination of shear fracture toughness are vital important for geology and geological engineering. Unfortunately there is not clear reply to a series of questions appeared in shear tests and fracture theories. For example, Is crack branching caused by shear fracture? Should shear fracture extend along original crack plane and does it can? Why predicted K_{IIC} by fracture theory is less than K_{IC}, calculated K_{IIC} based on shear tests was less than K_{IC} too? Should K_{IIC} be larger than or be less than K_{IC}? How proper shear fracture test should be conducted? etc. Based on theoretical study of stress state around crack tip under shear loading and comparison of maximum dimensionless tensile stress intensity factor with maximum dimensionless shear stress intensity factor this paper gives a clear answer to above-mentioned questions.

2 STRESS STATE AROUND CRACK TIP UNDER SHEAR LOADING

When a crack is loaded in shearing, as shown in Fig. 2, the stresses at the crack tip are expressed as[1]

$$\left.\begin{aligned}\sigma_r &= \frac{K_{II}}{\sqrt{2\pi\cdot r}}\sin\frac{\theta}{2}\left(1-3\sin^2\frac{\theta}{2}\right)\\ \sigma_\theta &= \frac{K_{II}}{\sqrt{2\pi\cdot r}}\left(-3\sin\frac{\theta}{2}\cos^2\frac{\theta}{2}\right)\\ \sigma_{r\theta} &= \frac{K_{II}}{\sqrt{2\pi\cdot r}}\cos\frac{\theta}{2}\left(1-3\sin^2\frac{\theta}{2}\right)\end{aligned}\right\}\quad(1)$$

where $K_{II}=\tau(\pi a)^{1/2}$ is shear stress intensity factor in original crack plane.

The dimensionless circumferential and shear stress intensity factors, f_θ and $f_{r\theta}$, can be derived from Eqn. (1) as follows:

$$\left.\begin{aligned}f_\theta &= \sigma_\theta\sqrt{2\pi\cdot r}/K_{II}\\ &= -3\sin\frac{\theta}{2}\cos^2\frac{\theta}{2}\\ f_{r\theta} &= \sigma_{r\theta}\sqrt{2\pi\cdot r}/K_{II}\\ &= \cos\frac{\theta}{2}\left(1-3\sin^2\frac{\theta}{2}\right)\end{aligned}\right\}\quad(2)$$

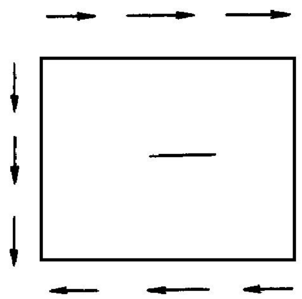

Fig. 2 Crack subjected to shearing

Differentiating (2) with respect to θ, maximum tensile and shear stress intensity factors could be obtained. Differentiation showed that the tensile stress intensity factor reaches its maximum at $\theta=-70.5°$ and has maximum value in dimensionless form equal to ±1.1546 (dimensionless tensile and compressive stress intensity factor) while shear stress intensity factor is equal to zero; shear stress intensity factor reaches its maximum value equal to 1 at $\theta=0$ (original crack plane) while circumferential stress intensity factor is equal to zero. Angular variation of circumferential and shear stress intensity factors, f_θ and $f_{r\theta}$, are shown in Fig. 3. Now question arises where the fracture will occur: at $\theta=0$ or at $\theta=-70.5°$?

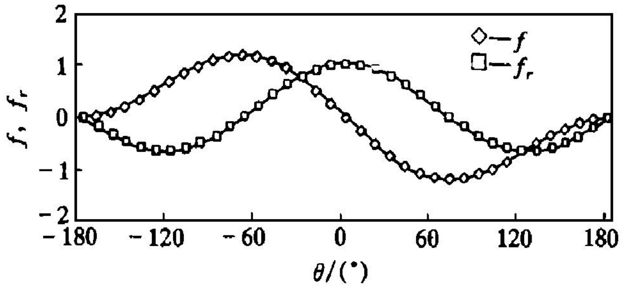

Fig. 3 Angular variation of circumferential and shear stress intensity factors in pure shear loading

Many fracture tests[2~4] under mode II loading proved that fracture was deviated from original crack plane with angle close to 70°. Fracture occurred at $\theta=70°$ not at $\theta=0°$ indicated that fracture was caused by tensile stress, not caused by shear stress. Thus fracture should be related to mode I (opening mode), could not be related to mode II. Recognizing the fracture at $\theta=-70.5°$ being mode I, the stress intensity factor of mode I in this plane under shear loading could be derived from σ_θ of Eqn. (1) as

$$K_I^{II}=1.1546\,\tau(\pi\cdot a)^{1/2}\quad(3)$$

where superscript II denotes mode II loading.

Then shear stress intensity factor at $\theta=0$ is expressed as

$$K_{II}=\tau(\pi\cdot a)^{1/2}\quad(4)$$

3 SHEAR MODEL

Fig. 3 shows when a crack is subjected by shear load, tensile stress intensity factor in dimensionless not only has maximum at $\theta=-70.5°$, but also it has value larger than 1 at angles $-90°<\theta<-50°$. To restrain or eliminate high tensile stress around crack tip at angles $\theta > |50|$, the compressive stress applied normal to crack plane is insufficient. For example, knowing tensile stress around crack tip based on numerical study of beam, Swartz and Taha[7] applied axial force during four point bending test, Petit[12] applied uniaxial compressive stress on a plate with central crack of 30°, crack branching was discovered again. Thus compressive stress parallel to crack plane should be applied in opposite direction above and below crack plane. This opposite compressive stress firstly serves as a shear stress on the crack plane.

In addition, Fig. 3 also shows that there is tensile stress intensity factor in dimensionless with lower value at small angles $\theta < |50|$. Thus compressive stress normal to crack plane with smaller magnitude should be applied to the specimen. Thus the shear model shown in Fig. 4 is proposed.

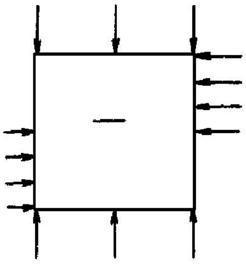

Fig. 4 Shear model

4 SHEAR FRACTURE TEST

The fixture of shear test is shown in Fig. 5. The rock type used in experiments is granite, marble and sandstone. The mechanical properties of these rocks are shown in Table 1.

Table 1 Mechanical properties of rocks

Rock type	σ_t /MPa	σ_c /MPa	φ /(°)	C^* /MPa	E /(GPa)	ν	K_{IC} /(MPa·m$^{1/2}$)
Marble *A*	3.1	79	41	25	25	0.30	1.26
Marble *B*	17.6	202	37	51	78	0.35	2.21
Granite	10.7	166	42	37	66	0.33	1.88
Sandstone	15.7	194	39	46	69	0.26	1.67

* C —Cohesion

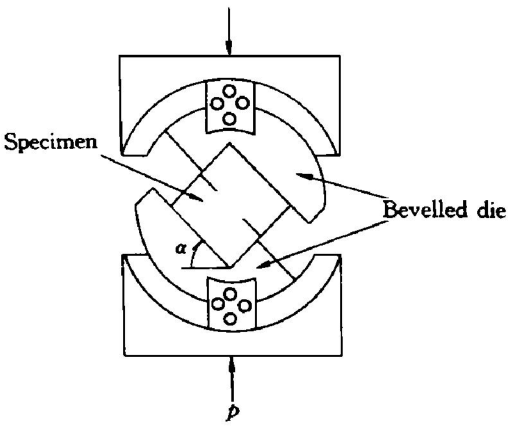

Fig. 5 Shear box test set up

Tested specimens dimensions are 70 mm × 70 mm × 35 mm, and 70 mm × 70 mm × 70 mm. The specimens with single or double notch cut by diamond saw were tested on a servo-controlled Instron 1342 press with a capacity of 100 kN, under position control.

The shear stress intensity factor for single notched specimens is expressed as[18]

$$K^{\sigma}_{IIC} = \frac{Q_m}{B\sqrt{W}} F\left(\frac{a}{W}\right) \tag{5}$$

where Q_m is maximum shear force on crack plane, $Q_m = p_m \sin\alpha$; p_m is maximum load; $F = [2.138 - 5.2(a/W) + 6.674(a/W)^2 - 3.331(a/W)^3] / \sqrt{1 - a/W}$ is the shape factor.

Considering the effect of normal compressive force on shear stress, the effective shear force, Q_{em}, is used instead of Q_m and expressed as

$$Q_{em} = p_m(\sin\alpha - \cos\alpha\tan\varphi) \tag{6}$$

where φ is internal friction angle, $\mu = \tan\varphi$.

The shear stress intensity factor for specimens with double notches is expressed as[19]

$$K_{IIC} = \frac{Q_{em}}{BW}\sqrt{\pi a} F\left(\frac{2a}{W}\right) \tag{7}$$

where

$$F\left(\frac{2a}{W}\right) = 1.780 + 3.095\left(\frac{2a}{W}\right) - 10.559\left(\frac{2a}{W}\right)^2 + 8.167\left(\frac{2a}{W}\right)^3$$

Tested results are given in Table 2. Dimensionless crack length (a/W), inclined angle, α, of tested specimens, number of specimens are also enclosed.

In Table 2 there are two figures of K_{IIC} for each rock type. The former is results of specimens with single notch, the latter for specimens with double notches. When inclined angle, α, of tested specimens is equal to or larger than 65°, shear fracture toughnesses, K_{IIC}, are basically equal to each other. Table 2 shows that shear fracture toughnesses for all rock types are larger than fracture toughness of mode I of corresponding rock. The ratio of K_{IIC}/K_{IC}, for marble *A* is about 3, for marble *B* is 2.7~2.8, for granite is close to 2.6, for sandstone is 2.7~3.2. In Table 2 the data of concrete tested by Ref. [14] were inserted, its ratio of K_{IIC}/K_{IC} is 1.6~1.8.

5 DISCUSSION

5.1 Critical review on maximum circumferential stress theory

The maximum circumferential tensile stress theory[2] states that crack extension begins when $\sigma_{\theta max}$ reaches a critical value as

$$I' = \cos\frac{\theta}{2}\left(\frac{K_I}{K_{IC}}\cos^2\frac{\theta}{2} - \frac{3}{2}\frac{K_{II}}{K_{IC}}\sin\theta\right) \tag{8}$$

For pure mode I loading, $K_{II} = 0$, and θ (fracture angle) = 0. $K_I = \sigma(\pi a)^{1/2} = K_{IC}$.

The theory let Eqn. (8) to be a parametric

Table 2 Parameters of mode II tests of rocks and concrete, K_{IIC} and ratio K_{IIC}/K_{IC}

Rock type	Marble A	Marble B	Granite	Sandstone	Concrete A[14]	Concrete B[14]
$K_{IC}/(\mathrm{MPa\cdot m^{1/2}})$	1.26	2.21	1.88	1.67	0.448	0.49
$K_{IIC}/(\mathrm{MPa\cdot m^{1/2}})$	3.73~4.26	6.01~6.20	4.87~4.85	4.56~5.26	0.802	0.78
a/W	0.47, 0.6	0.59, 0.68	0.6, 0.7	0.59, 0.71	0.6	0.6
$\alpha/(°)$	65, 70	70	65, 70, 75	70	70, 75, 80	65~80
Specimen number	8	6	34	6	8	11
K_{IIC}/K_{IC}	2.96~3.28	2.72~2.81	2.59~2.58	2.73~3.15	1.79	1.59

equation of a curve in K_I, K_{II} plane. It is correct from point of mathematical view. But it is incorrect from point of fracture mechanics view. It misleads into thinking that K_I, K_{II} are stress intensity factors at crack tip. Then attempt to predict ratio of K_{IIC}/K_{IC} followed: let $K_I = 0$, the ratio $K_{II}/K_{IC} = 0.866$ was calculated by Eqn. (8) at the fracture angle $\theta = -70.5°$. Then the theory simply makes $K_{II}/K_{IC} = K_{IIC}/K_{IC} = 0.866$.

Following a concept of a curve in K_I, K_{II} plane under mixed loading, other fracture theories, such as maximum energy release rate theory and minimum strain energy density theory, also considered that "the critical values of K_{IC} and K_{IIC} lie on a curve in the K_I, K_{II}-plane"[17], made similar mistakes: $K_{IIC} < K_{IC}$.

Note K_I, K_{II} under mixed loading in Eqn. (8) is specified to the loading modes I and II only. They are not stress intensity factors at crack tip. For example, K_{II} in the three equations of Eqn. (1) is used to specify loading mode II only. Under any loading the mode I and II stress intensity factors, K_I and K_{II}, should be expressed as[20]

$$\left.\begin{aligned} K_I &= \lim_{r\to 0}(2\pi r)^{1/2}\sigma_\theta \\ K_{II} &= \lim_{r\to 0}(2\pi r)^{1/2}\sigma_{r\theta} \end{aligned}\right\} \qquad (9)$$

It implies that shear stress intensity factor, K_{II}, could be derived from Equation of $\sigma_{r\theta}$ only.

Note differentiation of σ_θ in Eqn. (1) has the following relationship with $\sigma_{r\theta}$:

$$\frac{\partial \sigma_\theta}{\partial \theta} = -\frac{3}{2}\sigma_{r\theta} \qquad (10)$$

Let Eqn. (10) be equal to zero, the fracture angle of maximum dimensionless tensile stress intensity factor can be obtained, meanwhile shear stress, $\sigma_{r\theta}$, is equal to zero and K_{II} is equal to zero too. Thus after shear tests calculation of K_{II} by Eqn. (4) is incorrect, because at $\theta = -70.5°$, $K_{II} = 0$; at $\theta = 0$, there is no fracture.

How $K_{II} = 0.866K_{IC}$ should be interpreted? Substituting corresponding data into it gives $K_{II} = \tau(\pi\cdot a)^{1/2} = 0.866\sigma(\pi\cdot a)^{1/2}$, then it leads to $\tau = 0.866\sigma$.

It means that when mode I fracture under pure shear loading takes place at $\theta = -70.5°$, applied shear stress, τ, is only equal to 0.866σ (tensile stress in mode I loading). Substituting this value into Eqn. (3) yields $K_I^{II} = 1.1546\tau(\pi\cdot a)^{1/2} = 1.1546\times 0.866\sigma(\pi\cdot a)^{1/2} = \sigma(\pi\cdot a)^{1/2} = K_I$.

It implies that stress intensity factor of mode I under pure shear loading at $\theta = -70.5°$, K_I^{II} is equal to that of mode I under tensile loading, K_I. It proves again that fracture in pure shear loading is fracture of mode I. Thus the fracture toughness under shear loading should be calculated using Eqn. (3) instead of Eqn. (4).

It should be emphasized that all mixed mode theories are impossible to predict fracture toughness of mode II, K_{IIC}. The basic reason is that there is no existence of prerequisite for mode II fracture in their tension-shear loading mode, including pure shear loading (see below). Thus no shear fracture could occur. In addition, the prediction of the ratio, K_{IIC}/K_{IC}, by these fracture theories has only single value (although this value is different by different theory). It does not agree with the facts, because the ratio of K_{IIC}/K_{IC} for different materials is different.

5.2 Can pure shear fracture be obtained?

The answer is definite. To form shear fracture it is necessary to restrain tensile stress around crack tip induced by shear force, especially at $\theta = -70.5°$. The compressive stress normal to crack plane seems ineffective. The compressive stress parallel to crack plane, but in the opposite direction above and below crack plane, Fig. 4, is more effective to restrain this tensile stress. Note, this parallel compressive stress should be uniformly distributed on the lateral side of the block, instead of concentrated shear force.

Shah[4] conducted shear fracture tests on cylindrical tube specimen of 4340 steel subjected to torsion loading. Here fracture surface showed crack branching with angle 70°~75°. The "fracture toughness of mode II", equal to $74\ \mathrm{MPa\cdot m^{1/2}}$[4] based on crack initiation is less than fracture toughness of mode I, $K_{IC} = 80.4\ \mathrm{MPa\cdot m^{1/2}}$[4]. A new shear fixture[15], corresponding to shear model, Fig. 4, was used to conduct mode II fracture of 4340 steel. The experiments displayed fracture without branching and yielded fracture toughness of mode II equaled 139 MPa·

$m^{1/2}$, which is larger than that of mode Ⅰ. Note, the ratio of $K_{ⅡC}/K_{ⅠC}$ is equal to 1.73.

Izum, et al[14], RAO[19] and WANG[21~23] conducted tests on concrete and rocks using compression-shear box, corresponding to shear model, Fig. 4. They obtained co-planar fracture and fracture toughness of mode Ⅱ being larger than that of mode Ⅰ, Table 2.

5.3 How to judge fracture mode under any loading?

To judge the fracture mode means to judge what fracture mode, mode Ⅰ or mode Ⅱ, will occur under different loading. As mentioned above, fracture mechanics related fracture mode with loading mode, Fig. 1, did not pay sufficient attention to fracture mode. Considering fracture under shear loading as shear fracture, references and handbook only provided equations for shear stress intensity factor calculation at original crack plane. Thus many researchers[2~10] had to use these equations to calculate fracture toughness after shear tests, and led $K_{ⅡC}$ less than $K_{ⅠC}$. For example, after testing a large plate with central crack subjected by shear, fracture toughness calculated by Eqn. (4) will be certainly less than that calculated by Eqn. (3). Unfortunately up to now references and handbook have not provided the equations similar to Eqn. (3) to calculate fracture toughness of mode Ⅰ under shear loading at angle where tensile stress intensity factor has maximum value.

The study of fracture under shear loading indicated that analysis of stress state around crack tip (−180°~+180°) is vital important to judge fracture mode. Under any loading analysis of stress state at crack tip requires to define maximum circumferential tensile stress intensity factor and maximum shear stress intensity factor and to compare them. When maximum circumferential tensile stress intensity factor is larger than maximum shear stress intensity factor, only mode Ⅰ fracture could occur for brittle material. The ratio $f_{r\theta\max}/f_{\theta\max}$ is proposed as a parameter to do their comparison.

A prerequisite of possible occurance of mode Ⅱ fracture is maximum dimensionless shear stress intensity factor, $f_{r\theta\max}$, around crack tip should be larger than maximum dimensionless tensile stress intensity factor, $f_{\theta\max}$. But it is still insufficient for mode Ⅱ fracture. Fracture of mode Ⅱ could occur under given loading only if following inequality is satisfied: $f_{r\theta\max}/f_{\theta\max} > K_{ⅡC}/K_{ⅠC}$. Finally, fracture of mode Ⅱ begins when $K_{Ⅱ}$ of $\sigma_{r\theta\max}$ reaches a critical material constant value, $K_{ⅡC}$.

If above inequality is not satisfied, fracture of mode Ⅰ will certainly take place even if $f_{r\theta\max} > f_{\theta\max}$. A more simple prerequisite for mode Ⅰ fracture is $f_{r\theta\max}/f_{\theta\max} < 1$, it happens under many loading condition.

6 CONCLUSIONS

1) The fracture occurred under pure shear loading should be related to fracture of mode Ⅰ, should not be related to mode Ⅱ fracture. After shear testing the fracture toughness should be calculated by Eqn. (3).

2) Note, the mode of loading does not always correspond to the mode of fracture. When the mode of fracture under given loading is unclear, it could lead to miscalculate fracture toughness of material. The use of Eqn. (4) to calculate shear fracture toughness under shear loading leads to lower fracture toughness value is an extreme example.

3) To judge what fracture mode, mode Ⅰ or mode Ⅱ, will occur under different loading, one should study the stress state around crack tip (−180° ~ +180°) and find out maximum dimensionless circumferential stress intensity factor, maximum dimensionless shear stress intensity factor. Then according following criterion to judge fracture mode.

For mode Ⅱ fracture a prerequisite is:

$f_{r\theta\max}/f_{\theta\max} > 1$ and $f_{r\theta\max}/f_{\theta\max} > K_{ⅡC}/K_{ⅠC}$.

If above inequality is not satisfied, fracture of mode Ⅰ will certainly take place. A more simple prerequisite for mode Ⅰ fracture is $f_{r\theta\max}/f_{\theta\max} < 1$, it happens under many loading condition.

If the prerequisite for mode Ⅰ or mode Ⅱ is satisfied, the criteria for crack extension initiation of corresponding mode Ⅰ or mode Ⅱ are

$K_{Ⅰ} = K_{ⅠC}$ or $K_{Ⅱ} = K_{ⅡC}$.

4) The maximum circumferential stress theory and other fracture theories predicted ratio $K_{ⅡC}/K_{ⅠC}$ being less than 1 is incorrect. Actually, all mixed mode theories are impossible to predict mode Ⅱ fracture toughness due to inexistence of prerequisite for mode Ⅱ fracture in tension-shear loading (including pure shear loading). Because $f_{r\theta\max}/f_{\theta\max} < 1$, the mode Ⅱ fracture could not occur.

5) Applied side compressive stress, the idea of Fig. 4, is more important to restrain tensile stress around crack tip and makes a favorable situation to induce co-planar fracture in mode Ⅱ.

6). The pure shear loading applied to crack surface is more dangerous than tensile loading, because the applied stress to initiate crack extension under shear loading is less than that under tension.

7) Fracture toughness of mode Ⅱ is larger than that of mode Ⅰ, if experiment is conducted according to a prerequisite for mode Ⅱ fracture. The ratio, $K_{ⅡC}/K_{ⅠC}$, is different for different materials.

[REFERENCES]

[1] Lawn B R and Wilshaw T R. Fracture of Brittle Solids

[M]. London: Cambridge University Press, 1975.

[2] Erdogan F and Shi G C. On the crack extension in plates under plane loading and transverse shear [J]. J Basic Eng, 1963, 85(4): 519- 527.

[3] Ewing P D and Williams J G. The fracture of spherical shells under pressure and circular tubes with angled crack in torsion [J]. Int J Fracture, 1974, 10(4): 537- 544.

[4] Shah R C. Fracture under combined modes in 4340 steel [J]. ASTM STP 560, 1974. 29- 52.

[5] Ingraffea A R. Mixed mode fracture initiation in Indiana limestone and westerly granite [A]. Einstein H H. Rock Mechanics From Research to Application [C]. Proc 22nd US Symp on Rock Mech, Cambridge, Mass, 1981, 199 - 204.

[6] HUANG J and WANG S. An experimental investigation concerning the comprehensive fracture toughness of some brittle rocks [J]. Int J Rock Mech Min Sci & Geomech Abstr, 1985, 22(2): 99- 104.

[7] Swartz S E and Taha N M. Mixed mode crack propagation and fracture in concrete [J]. Eng Fract Mech, 1990, 35: 1- 3.

[8] XU Dao-yuan and FU Xiao-lin. The study of determination of mixed mode fracture criterion on concrete by four point bending specimens [J]. Chinese J of Water Conservancy, (in Chinese), 1984, 9.

[9] GAO Hua, WANG Zhi-qiang, YANG Chen-shou, et al. An investigation on the brittle fracture of K_{I}, K_{II} composite mode cracks [J]. Acta Metallurgica Sinca, (in Chinese), 1979, 15(3): 380- 391.

[10] Richard H A. Some theoretical and experimental aspects of a mixed mode fracture [A]. Valluri S R. Advances in Fracture Research [C]. 6th Int Conf on Fract, 1984. 3337- 3344.

[11] Watkins J and Liu K L W. A finite element study of the short beam test specimen under mode II loading [J]. Int J of Cement Composites and Lightweight Concrete, 1985, 7(1): 39- 47.

[12] Jean-Pierre Petit. Can natural faults propagate under mode II conditions? [J]. Tectonics, 1988, 7(6): 1243 - 1256.

[13] Melin S. When does a crack grow under mode II conditions? [J]. Int J Fracture, 1986, 30: 103- 114.

[14] Izumi M, Mihashi H and Nomura N. Fracture toughness of concrete for mode II [A]. Wittman F H. Fracture Toughness and Fracture Energy of Concrete [C]. Amsterdam: Elsevier Science Publishers, 1986. 347- 354.

[15] Tsangarakis N. Mode II fracture toughness of 4340 steel [J]. Engineering Fracture Mechanics, 1985, 22 (4): 617- 624.

[16] Hussain M A, Pu S L and J Underwood. Strain energy release rate for a crack under combined mode I and mode II [J]. ASTM STP 560, 1974, 2- 28.

[17] Sih G C. Strain energy density factor applied to mixed mode crack problems [J]. Int J Fract, 1974, 10(3): 305- 321.

[18] YU Yao-zhong, QIAO Chang-xin and ZHOU Qun-li. Fracture Mechanics of Rock and Concrete [M]. Changsha: Central South University of Technology Press, 1991.

[19] RAO Q H. Pure Shear Fracture of Brittle Rock, a Theoretical and Laboratory Study [D]. Lulea: Lulea University of Technology, 1999.

[20] Ichikawa M and Tanaka S. A critical analysis of the relationship between the energy release rate and the stress intensity factors for non-coplanar crack extension under combined mode loading [J]. Int J Fracture, 1982, 18 (1): 19- 28.

[21] CHANG K J. On the maximum strain criterion —a new approach to the angled crack problem [J]. Eng Fract Mech, 1981, 14: 107- 124.

[22] WANG Gui-yao. Study of Mode II (Shear) Fracture of Rock and its Engineering Application [D]. Changsha: Central South University of Technology, (in Chinese), 1996.

[23] WANG Gui-yao, SUN Zong-qi and XU Ji-cheng. Test method for determining mode II fracture toughness of rock [J]. The Chinese Journal of Nonferrous Metals, (in Chinese), 1999, 9(1): 175- 179.

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