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Simulating of marble subjected to uni-axial loading using index-parabola damage model ①

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[Abstract] The limitations of several existing classical rock damage models were critically appraised. Thereafter, a description of a new model to estimate the response of rock was provided. The results of an investigation lead to the development and confirmation of a new index-parabola damage model. The new model is divided into two parts, fictitious damage and real damage and bordered by the critical damage point. In fictitious damage, the damage variable follows the index distribution, while in the real damage a parabolic distribution is used. Thus, the so-called index-parabola damage model is derived. The proposed damage model is applied to simulate the damage procedure of marble under uni-axial loading. The results of the tests show that the proposed model is in excellent agreement with experimental data, in particular the nonlinear characteristic of rock deformation is adequately represented.

[Key words] index-parabola damage model; uni-axial loading; nonlinear behavior of marble

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1 INTRODUCTION

Rock is a highly nonlinear medium, whose behavior can be categorized into an initial linear response followed by plastic or the weakened phase after attaining the peak stress[1]. Traditionally, the average modulus of rock is taken as 50% of elastic modulus of the peak stress in traditional calculations. However, elastic modulus of rock varies during the course of the loading history. This property is a function of time, and can represent the time-dependent damage behavior of rock during loading. Therefore, it is unreasonable to expect such an approach to handle practical problems.

It is generally accepted that certain mathematical models may be introduced to study rock damage behavior. But most of the existing models are oversimplified and their applications are limited to certain conditions. At present, the study of nonlinear damage and the correct description of damage have become the focus of research into rock damage subjected to uni-axial loading. In this paper a new damage model, the so-called index-parabola damage model, is proposed and presented, and is used to simulate the damage of marble under uni-axial loading. Test results are provided to verify the technique.

2 INDEX-PARABOLA DAMAGE MODEL

In the study of rock mechanics, damage constitutes an important area, which has been largely neglected in the past. Thus, it is essential to understand the characteristics of damage. In general, brittle behavior is a characteristic of rock and its damage procedure is inevitably accompanied by brittle damage fracture. Damage has the decisive effect on the deformation of rock, and it accounts for most of the deformation under loading. Damage can be considered as a continuous process, and on the basis of the discussion mentioned above, many researchers have proposed various damage models to describe rock damage.

The Loland damage model[2] divides the damage extension into two areas. Each one is described by a different function. In the model, the crack arises and extends in the element only and keeps in a tiny local area before attaining peak stress. Most of the cracks extend unsteadily in the damage area after the peak stress. This model predicts accurately experimental results before the peak stress. It assumes that the stress—strain, $\sigma-\varepsilon$, relationship is linear after the peak stress, but in fact the relation is not linear. Thus, it is an approximation of a complex behavior.

Mazars[3] damage model is also divided the stress—strain curve into two parts, defined as the section before and after the peak stress. In this model, the $\sigma-\varepsilon$ curve deviates slightly from test results before attaining the peak stress. However, it may be regarded as a linear relationship. In this case, there is no initial damage ($D = 0$). After the stress reaches the peak, the strain increases but the stress decreases

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according to a function after the peak stress; the behavior corresponds to the formation of macro-cracks coupled with rapidly unsteady extension. At this point, the stiffness of the rock decreases sharply. Clearly, this model is an approximation because of the linear assumption made at the initial stage of the response phase.

Sidoroff damage model[4] is based on the equivalent principle. The basis of the model is anisotropy theory, which is suitable for brittle-elastic materials. As before, this model assumes that there is no damage before the peak stress. The Bilinear model[5] considers a linear stress—strain relationship for the response before and after the peak stress. Thus, there is no damage before attaining the peak stress and the response thereafter is linear.

It can be observed that there is commonality for the damage models mentioned above. In all the aforementioned models, the damage is zero or exhibit a linear relationship before or after the peak stress. Other damage models also exist, some of which are Krajcinovic damage model[6], Positive Anisotropy damage model[7] and the Creep damage model[7]; most of these are often applied to more complex three-dimensional damage situations.

In order to overcome the deficiencies of the above models and to better reflect the nonlinear characteristic of damage, an Index-Parabola damage model (IPDM) to describe behavior of rock subjected to uni-axial loading is proposed. In this approach, the notion of fictitious damage is introduced. Further, the damage procedure is divided into fictitious damage and real damage. For the case of the strain condition $\varepsilon < \varepsilon_D$, where ε_D corresponds to the critical damage point, it is assumed as the fictitious damage stage, in which the damage variable D follows the index distribution. This is consistent with the first stage response of the Loland damage model. When $\varepsilon > \varepsilon_D$, the damage procedure requires additional input. At this stage, the damage variable D follows a parabola distribution. Both are nonlinear and a typical structure of the model is shown in Fig. 1.

The equation of damage variable D is given by

$$D = \begin{cases} 1 - a\varepsilon^b & \varepsilon < \varepsilon_D \\ a'\varepsilon^2 + b'\varepsilon + c & \varepsilon \geqslant \varepsilon_D \end{cases} \tag{1}$$

where a, b, a', b', c are material constants and are obtained experimentally. The a' determines the concavity of the parabola. Clearly, the parabola exhibits a downward (parts of the imaginary line) as well as an upward trend.

The model considers both the strain, ε, and time, t, as independent variables. Both parameters reflect the time-dependent property of the procedure. However, the second stage of rock damage exhibits a very rapid deterioration for brittle materials, and the response is difficult to capture. Nevertheless, it is more convenient to express the damage with strain. The main advantage is that it represents the nonlinear property of rock deformation adequately and is similar to actual behavior.

3 EXPERIMENTAL VERIFICATION AND APPLICATION

A series of uni-axial experiments were designed and conducted to verify the index-parabola damage model described above. However, in such experiments, it is not possible to measure material damage parameters in conventional experimental equipment. This is one of the main reasons that constrains the wider use of damage mechanics in engineering[8]. For the present investigation, the experiment was undertaken on an advanced Instron servo-hydraulic test machine, the rock used was a block of marble which was collected along a highway and brought to the laboratory for preparing the circular samples of d50 mm × 100 mm.

The cylindrical samples were subjected to uni-axial compression. The results indicate a high compressive strength of the marble. For this reason, the idealized stress—strain curves were obtained under load control condition. The equipment permits direct random control of loading rate and loading path by changing parameters in the function generator and in X-Y automatic recorder of the system. The experimental response curves were automatically recorded.

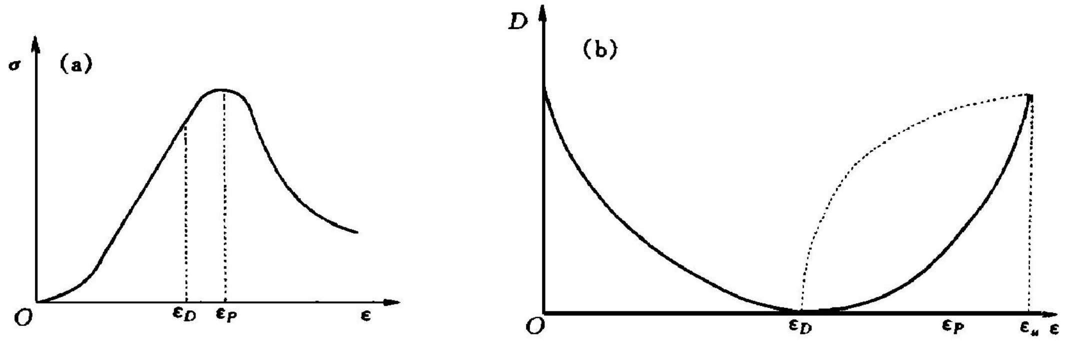

Fig. 1 Index-parabola damage model
(a) —Parabola; (b) —Index

Load-displacement curves for samples 1, 2 and 3 are shown in Fig. 2.

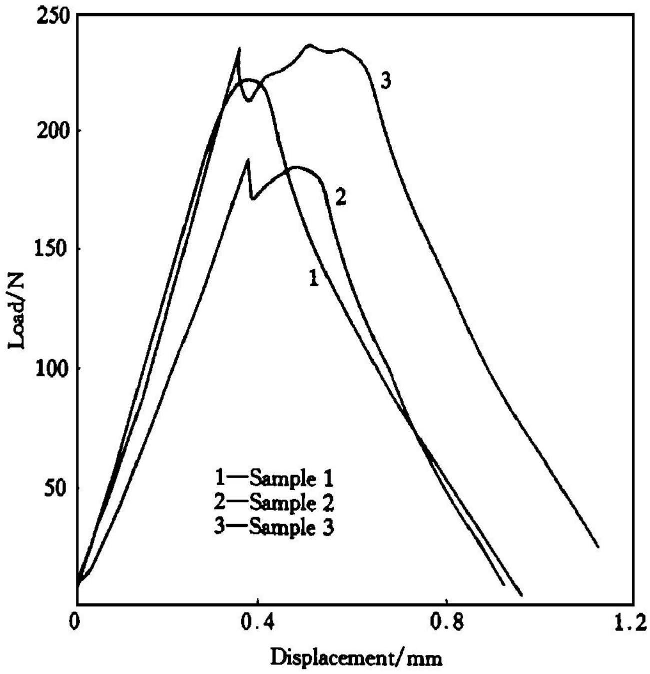

Fig. 2 Idealized load-displacement curves of marble under load control
(loading rate is 500 kN/ 1 000 s)

Many ways can be found to express and measure the damage variable, but the most representative mechanical parameter to betterly reflect damage is elastic modulus. Since the damage procedure is always changing with elastic modulus, it is realistic to study the time-dependent phenomena of elastic modulus. By measuring the change of elastic modulus, the damage variable can be indirectly obtained; which can similarly also be acquired by the elastic eigen-constitutive equation. Ref. [9] discusses in detail the effective modulus method in damage mechanics of rock.

In this investigation, the damage variable D is written as

$$D = 1 - E(\varepsilon)/E_0 \tag{2}$$

where D is the damage variable and $E(\varepsilon)$ is the effective elastic modulus. These parameters correspond to each step of the loading regime. E_0 is the elastic modulus of the rock before damage.

In order to analyze the data and to conduct a more convenient analysis, we regard the peak elastic modulus as E_0. The relationship between load and displacement shown in Fig. 2 has been transformed into curves relating elastic modulus, E, to changing values of displacement, u.

Since the strain is defined as $\varepsilon = u/L$, where L is length of the sample, the plots of Figs. 3, 4 represent the relationship between elastic modulus, E, and strain, ε. In Figs. 3, 4, the dotted line represent the response predicted by the Index-Parabola damage model. To eliminate the effect of initial loading and the effect from the samples themselves, the starting point was taken at $u = 0.04$ mm. Calculations derived from Eqn. (2) are shown in Table 1, and from which the damage time-dependent response is obtained.

It can be shown from the analysis that the critical damage point of sample 2 and 3 is adjacent to the location of the peak stress. At this point, the samples

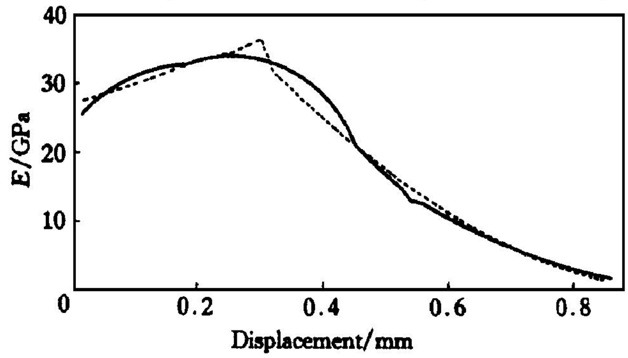

Fig. 3 Elastic modulus change curves of sample No. 1

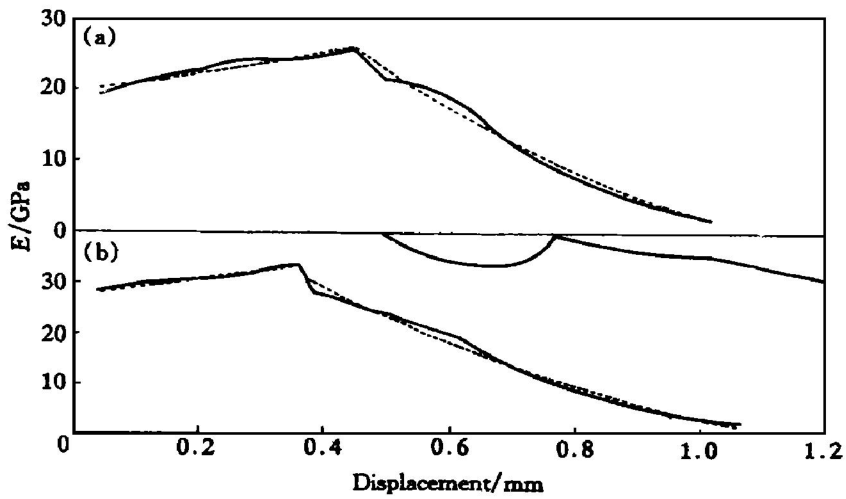

Fig. 4 Elastic modulus change curves
(a) —Sample 2; (b) —Sample 3

Table 1 Experimental results

Sample No.	Corresponding figure in text	Predicted curve (function)	Critical damage point u_D /mm	Compressive strength /MPa	Peak elastic modulus /GPa
1	Fig. 3	$E=26.686\,e^{-1.0912u}$, $E=67.288u^2-135.65u+68.477$	0.30	113.46	33.84
2	Fig. 4(a)	$E=19.481\,e^{-0.7411u}$, $E=44.281u^2-105.23u+59.291$	0.38	96.52	25.70
3	Fig. 4(b)	$E=27.711\,e^{-0.515u}$, $E=27.144u^2-81.713u+57.634$	0.36	120.65	31.74

exhibit brittle behavior. Nevertheless, the predicted curves are in good agreement with experimental data.

Table 1 shows that the Index-Parabola damage model is an appropriate and reasonable tool to simulate the damage procedure of marble under uni-axial loading. Using this model, it is possible to express the creep life and the damage time of marble by the critical damage point. It is also reasonable to use the technique to build up the damage constitutive equation suitable for actual damage of rock in practice arising from blasting. Ref.[9] presented by curve for another type of rock, Andesite. The details are typical of the Index-Parabolic curve.

4 CONCLUSIONS

1) The limitations of several classical rock damage models are considered and analyzed. Many of the models are derived from simplification and approximation of behavior.

2) A new Index-Parabola damage model is proposed and presented. The idea of fictitious damage is introduced. The model is divided into two parts: fictitious damage and real damage, which are bordered by the critical damage point.

3) The damage variable follows the index distribution in the fictitious stage and a parabolic distribution in the real damage stage. In this way, it can be applied to estimate the damage procedure of marble under uni-axial loading.

4) Results of tests on marble have shown that the model is in excellent agreement with experiments. The model is able to represent the nonlinear characteristic of rock deformation with great accuracy.

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