[Article ID] 1003- 6326(2001)02- 0311- 04

Fitting method of pseudo-polynomial for solving nonlinear parametric adjustment①

TAO Hua-xue(陶华学), GONG Xiu-jun(宫秀军), GUO Jin-yun(郭金运)
(Institute of Geo-Science and Technology, Shandong University of Science and Technology, Taian 271019, P. R. China)

[Abstract] The optimal condition and its geometrical characters of the least-square adjustment were proposed. Then the relation between the transformed surface and least-squares was discussed. Based on the above, a non-iterative method, called the fitting method of pseudo-polynomial, was derived in detail. The final least-squares solution can be determined with sufficient accuracy in a single step and is not attained by moving the initial point in the view of iteration. The accuracy of the solution relys wholly on the frequency of Taylor's series. The example verifies the correctness and validness of the method.

[Key words] nonlinear parametric adjustment; fitting method of pseudo-polynomial; transformed surface
[CLC number] TB22 **[Document code]** A

1 OPTIMAL CONDITION AND ITS GEOMETRICAL CHARACTERS

The adjustment model with n observation and $m(m < n)$ parameters may be written as

$$\left.\begin{array}{l} l^r = y^r(x^a) + e^r \\ e^i \sim N(0, g^{ij}) \end{array}\right\} \tag{1}$$

where $l^r(r = 1, 2, \cdots, n)$ represents components of observations; $e^r(r = 1, 2, \cdots, n)$ represents components of error; $y^r(u^a)(a = 1, 2, \cdots, m)$ is assumed to be a nonlinear map from unknown parametric sets $\{x^a\}$ to components of adjusted values; e^i is subjected to be a normal distribution and $g^{ij} = E(l^i \cdot l^j) = E(e^i \cdot e^j)$ is the variance-covariance of error vector.

With the method of nonlinear least-squares adjustment, the minimum of Eqn. (2) can be sought:

$$F(x) = e^r(x) g_{rs} e^s(x) \tag{2}$$

It is known that a necessary condition for extreme value of a general continuous and differentiable multivariate function $\Phi(X)$ is $\nabla\Phi(\bar{X}) = 0$, it is to say that $\bar{X}$ is a stable point, however it is not sufficient. One of its sufficient conditions is that the Hessien matrix of $\bar{X}$ is positive-defined. The Taylor's series expansion of Eqn. (2) evaluated at $\bar{X}$ is represented as

$$F(x) = F(\hat{x}) + F_a(\hat{x})\Delta x^a + F_{a\beta}(\hat{x})\Delta x^a \Delta x^\beta / 2 + \cdots$$

The above condition of multivariate function's extreme value was applied to $F(x)$[8], then

$$e^r(\hat{x}) g_{rs} A^s_a(\bar{X}) = 0 \tag{3a}$$

and

$$K_n \| e(\bar{X}) \| < 1 \tag{3b}$$

where

$$A^s_a(\hat{x}) = \frac{\partial y^r}{\partial x^a}(\hat{x}), \quad \Omega_{a\beta} = \frac{\partial y^r}{\partial x^a \partial x^\beta}(\hat{x}),$$

$$\Delta x^a = x^a - \hat{x}^a, \quad K_n = \mathrm{II}/\mathrm{I},$$

$$\mathrm{I} = A^r_a(\hat{x}) g_{rs} A^s_\beta(\hat{x}) \Delta x^a \Delta x^\beta,$$

and

$$\mathrm{II} = \frac{e^r(\hat{x})}{\| e(\hat{x}) \|} g_{rs} \Omega^s_{a\beta} \Delta x^a \Delta x^\beta$$

Eqn. (3) is the sufficient and necessary condition of seeking the extreme value of Eqn. (2). It has a direct geometrical senses: Eqn. (3a) means that the residual vectors are orthogonal to the tangent space at the extreme point; Eqn. (3b) means that the extreme point lies in the circle whose center is y and whose radius is l/K_n.

2 TRANSFORMED SURFACE AND LEAST-SQUARES ADJUSTMENT

Iterative method was often used in solving the nonlinear parametric adjustment. To some extent, this method improves the accuracy of the adjusted results, but it is difficult to assess some characters of the resolution in the whole. However it shows us an important inspiration from the progress of solving the model: the point which minimizes the sum of squares derivations must be a stable point. In the view of differential geometry, the residual vector should be orthogonal to tangent surface at the stable point. Based on the above condition, we construct a hyper-surface, called Q-surface. This surface satisfies the following conditions:

1) It comes through the observational point Q.

① [Foundation item] Project (49874001) supported by the National Natural Science Foundation of China
[Received date] 1999- 12- 03; [Accepted date] 2000- 09- 25

2) A point in the Q-surface is generated by intersection of the normal plane and the line containing the point Q, and this line is orthogonal to the normal plane.

So the initial point moves in the model surface, the corresponding point in the hyper surface would accordingly vary. When the varied point in the hyper surface coincides with the point Q, the corresponding point $\bar{P}$ in the model surface is the adjusted point.

The characters of the hyper-surface.

1) Construction of the Q-surface

As shown in Fig. 1, based on the initial point P, the dimensions of the tangent space of the model at the point P is m, and the dimensions of the normal space is $n-m$. Because there are $n-(n-m)=m$ lines which are orthogonal to the normal space through point Q in the n dimensional space, the hypersurface is also m dimensional manifold. For example, when the model surface is the two-dimensional manifold (sphere) embedded in three-dimensional space, the dimensions of the tangent plane through random point P are two. There is a one-dimensional normal line through this point and there is a two-dimensional plane crossing the point Q that is orthogonal to the normal line. When the selected point P varies along the sphere, the track of the point P' is also a sphere.

2) The hyper-surface is also continuous and differentiable.

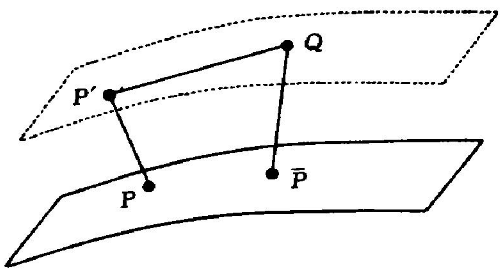

Fig. 1 Q-surface and parametric adjustment

3 PROCEDURE OF SOLVING ADJUSTED MODEL

Through the above analysis, it is known that the hyper surface Q plays an important role in the adjustment. When point P in the model surface moves in the model surface, the corresponding point P' in the Q-surface would varies. Conversely, When the point P' in the Q-surface moves along the Q-surface, the corresponding point P in the model surface would varies. So the vector $P\bar{P}$ and $P'Q$ would vary accordingly, as shown in Fig. 2.

Write $P\bar{P}$ as $\{\bar{y}'^r - y^r\} = \{\Delta y^r\}$ and $P'Q$ as $\{y^r_Q - y'^r\} = \{\Delta y'^r\}$, where $\{y^r\}$ is the function of $\{y'^r\}$ and $\{\Delta y'^r\}$, which is related to $\{\Delta y^r\}$.

Let

$$\Delta y^r = \Delta y^r(\Delta y'^s) \tag{4}$$

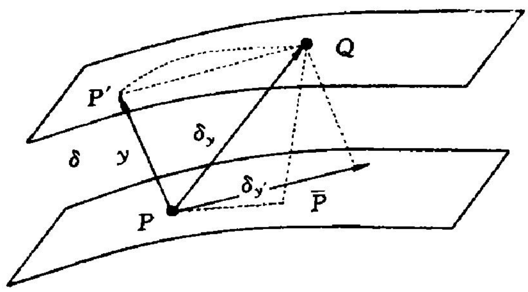

Fig. 2 Construction of Q-surface

So if the obvious functional relation of Eqn. (4) is found, the direct method of solving adjustment model would appear. Then it would be resolved by constructing a pseudo-polynomial.

Before commenting further, the notational conventions will be introduced as

1) Convention of the upper/lower indices

The lower Roman letters r, s, …, vary from 1 to n; the upper Roman letters L, R, …, vary from $m+1$ to n; the lower Greek letters α, β, …, vary from 1 to m.

2) The coordinate of the points in the Q-surface is expressed as $\{\tilde{y}^r\}$ (point Q as $\{y^r_Q\}$) and the coordinates of the points in model surface as $\{y^r\}$ (point $\bar{P}$ as $\{\bar{y}^r\}$). Write "Δ" before the coordinate to indict the coordinate difference of the points in the same surface and "δ" before the coordinate to indict the coordinate difference of the points in the different surface.

We solve model (1) by four steps as following.

3.1 Three transformed expressions of model surface[8]

Firstly, the equation of the model surface is

$$y^r = y^r(x^\alpha) \tag{5}$$

Expand Eqn. (5) into Taylor's series at initial value x^α_0 as

$$y^r = y^r(x^\alpha_0) + A^r_\alpha \Delta x^\alpha + \Omega^r_{\alpha\beta}\Delta x^\alpha \Delta x^\beta/2 + \Phi^r_{\alpha\beta\gamma}\Delta x^\alpha \Delta x^\beta \Delta x^\gamma/6 + \cdots \tag{6}$$

where $\Delta x^\alpha = x^\alpha - x^\alpha_0$; $A^r_\alpha = \dfrac{\partial y^r}{\partial x^\alpha}(x^\omega_0)$; $\Omega^r_{\alpha\beta} = \dfrac{\partial^2 y^r}{\partial X^\alpha \partial x^\beta}(x^\omega_0)$ and $\Phi^r_{\alpha\beta\gamma} = \dfrac{\partial^3 y^r}{\partial x^\alpha \partial x^\beta \partial x^\gamma}(x^\omega_0)$.

Eqn. (6) corresponds to the Gauss-form of the model surface.

In Eqn. (6), a subset of m equations is divided into a group and a subset of remaining $n-m$ equations is divided into another group, which are respectively

$$y^\mu = y^\mu(x^\alpha) \tag{7a}$$

$$y^L = y^L(x^\alpha) \tag{7b}$$

If $J = \left|\left(\dfrac{\partial y^\mu}{\partial x^\alpha}\right)_{m\times m}\right| \neq 0$, express $\{x^\alpha\}$ by $\{y^\mu\}$ as

$$x^\alpha = x^\alpha(y^\mu)$$

$$= x^a(y_0^\mu) + R^a_\lambda \Delta y^\lambda + \Lambda^a_{\lambda k}\Delta y^\lambda \Delta y^K/2 + \Theta^a_{\lambda k\delta}\Delta y^\lambda \Delta y^k \Delta y^\delta/6 + \cdots \tag{8a}$$

and

$$y^L = y^L(x^a) = y^L(x^a(y^\mu)) = f^L(y^\mu) \tag{8b}$$

where $\Delta y^\lambda = y^\lambda - y_0^\lambda$, $R^a_\lambda = \frac{\partial x^a}{\partial y^\lambda}(y_0^\omega)$, $\Lambda^a_{\lambda k} = \frac{\partial^2 x^a}{\partial y^\lambda \partial y^K}(y_0^\omega)$ and $\Theta^a_{\lambda k\delta} = \frac{\partial^3 x^a}{\partial y^\lambda \partial y^k \partial y^\delta}(y_0^\omega)$.

Eqn. (8) is equivalent to the Monge form of the model surface.

Substitute Eqn. (8) for Eqn. (7b), it can be gotten:

$$N^L = y^L - f^L(y^\mu) = 0 \tag{9}$$

which is the functional form of the model surface.

The expression of each order derivates and their relations can be derived.

Let $A^r_a = \{A^\rho_a, A^R_a\}$; $\Omega_{a\beta} = \{\Omega^\rho_{a\beta}, \Omega^R_{a\beta}\}$ and $\Phi_{a\beta\gamma} = \{\Phi^\rho_{a\beta\gamma}, \Phi^R_{a\beta\gamma}\}$. The differentiation of the both sides of Eqn. (9) with respect to $\{y^r\}$ is[8]

$$N^L_r = \frac{\partial N^L}{\partial y^r} = \{N^L_\rho, N^L_R\} = \left\{-\frac{\partial y^L}{\partial y^\rho}, \delta^L_R\right\} \tag{10}$$

The further differentiation of the both sides of Eqn. (10) with respect to $\{y^r\}$ is

$$\left.\begin{aligned} N^L_{\rho\mu} &= \frac{\partial^2 N^L}{\partial y^\rho \partial y^\mu} = -\frac{\partial^2 y^L}{\partial y^\rho \partial y^\mu} \\ &\text{and} \\ N^L_{\rho\mu\tau} &= \frac{\partial^3 N^L}{\partial y^\rho \partial y^\mu \partial y^\tau} = -\frac{\partial^3 y^L}{\partial y^\rho \partial y^\mu \partial y^\tau} \end{aligned}\right\} \tag{11}$$

If adopt $\{y^a\}$ as the coordinates of the model surface, the Gauss-form of the model surface is written as $y^r = h^r(y^a)$ and the partial derivates with respect to new coordinate system are listed as

$$\frac{\partial y^r}{\partial y^a} = \left\{\frac{\partial y^\rho}{\partial y^a}, \frac{\partial y^R}{\partial y^a}\right\} = \{\delta^\rho_a, -N^R_a\} \tag{12a}$$

$$\frac{\partial^2 y^r}{\partial y^a \partial y^\beta} = \{0, -N^R_{a\beta}\} \tag{12b}$$

$$\frac{\partial^3 y^r}{\partial y^a \partial y^\beta \partial y^\gamma} = \{0, -N^R_{a\beta\gamma}\} \tag{12c}$$

Due to the deriving rules of the compound function that is

$$\frac{\partial y^r}{\partial y^k} = \frac{\partial y^r}{\partial y^\lambda} \cdot \frac{\partial y^\lambda}{\partial y^k} \tag{13}$$

Substitute Eqn. (13) with the above outcomes $\{\delta^\rho_K - N^R_K\} = \{A^\rho_\lambda, A^R_\lambda\} R^\lambda_K$, then

$$A^a_\lambda R^\lambda_k = \delta^a_k \tag{14a}$$

and

$$N^L_k = A^L_\lambda R^\lambda_k \tag{14b}$$

The further differentiations of Eqn. (13) are

$$A^a_{k\tau} = -R^\lambda_\mu \Omega^\mu_{\beta\gamma} R^\beta_k R^\gamma_\tau \tag{15a}$$

$$N^L_{k\tau} = -(N^L_\mu \Omega^\mu_{\beta\gamma} + \Omega^L_{\beta\gamma}) R^\beta_k R^\gamma_\tau \tag{15b}$$

Similarly, $\Theta^\lambda_{k\tau\omega}$ and $N^L_{k\tau\omega}$ can be obtained.

3.2 Computing $\{\tilde{y}^r\}$

As we know that $\{\delta\tilde{y}\} = \{\tilde{y} - y\}$ is orthogonal to the model surface, it must be expressible as a linear combination of the $n - m$ gradient-vectors as

$$\delta\tilde{y}_s = N^M_s C_M \tag{16}$$

On the other hand, $\{\delta y'\}$ is orthogonal to gradient vector, so

$$N^L_r \delta y'^r = 0 \tag{17}$$

Substitute Eqn. (17) with $\delta y' = \delta y - \delta\tilde{y}$ and $\delta\tilde{y}^r = g^{rs}\delta\tilde{y}_s$, then $N^L_r g^{rs} N^M_s C_M = N^L_r \delta y^r$ was gotten.

The resolutions are as follows[8]:

$$C_K = Q_{KL} N^L_m \delta y^m \tag{18a}$$

$$\delta\tilde{y}^r = g^{rs} N^K_s C_K \tag{18b}$$

where $Q_{KL} B^{LM} = \delta^M_K$ and $B^{LM} = N^L_r g^{rs} N^M_s$.

3.3 Construction of pseudo-multinomial

From Fig. 2, $\delta y^r = y^r_Q - y^r$ and $\delta\tilde{y}^r = \tilde{y}^r - y^r$ can be found, among which m variables that can be written as $\delta y^a = y^a_Q - y^a$ and $\delta\tilde{y}^a = \tilde{y}^a - y^a$ is selected. In order to construct the pseudo-multinomial, it can be proved that $\{\tilde{y}^a\}$ is the function of $\{y^a\}$.

Let $\Delta y^a = \tilde{y}^a - y^a$ and $\Delta\tilde{y}^a = y^a_Q - \tilde{y}^a$, then $\{\Delta y^a\}$ must be the function of $\{\Delta y^a\}$. Again let

$$\tilde{A}^a_\beta = \frac{\partial \tilde{y}^a}{\partial y^\beta} = \delta^a_\beta + \frac{\partial \delta\tilde{y}^a}{\partial y^\beta} \tag{19a}$$

$$\tilde{\Omega}^a_{\beta\gamma} = \frac{\partial^2 \tilde{y}^a}{\partial y^\beta \partial y^\gamma} = \frac{\partial^2 \delta\tilde{y}^a}{\partial y^\beta \partial y^\beta} \tag{19b}$$

Then we can get

$$\left.\begin{aligned} \frac{\partial \Delta\tilde{y}^a}{\partial y^\beta} &= \frac{\partial(y^a_Q - \tilde{y}^a)}{\partial y^\beta} = -\tilde{A}^a_\beta \\ &\text{and} \\ \frac{\partial^2 \Delta\tilde{y}^a}{\partial y^\beta \partial y^\gamma} &= -\tilde{\Omega}^a_{\beta\gamma} \end{aligned}\right\} \tag{20}$$

We must calculate the inverse partial derivative to obtain $\{\Delta y^a\}$ which is a function of $\{\Delta\tilde{y}^a\}$. Let $\tilde{R}^a_\beta = \frac{\partial y^a}{\partial \tilde{y}^\beta}$ and $\tilde{\Lambda}^a_{\beta\gamma} = \frac{\partial^2 \tilde{y}^a}{\partial y^\beta \partial y^\gamma}$. The similar with the conclusion of Eqns. (14) and (15), the following equation can be gotten:

$$\tilde{A}^a_\beta \tilde{R}^\beta_k = \delta^a_k \tag{21a}$$

and

$$\tilde{\Lambda}^a_{\beta\gamma} = -\tilde{R}^a_\mu \tilde{\Omega}^\mu_{\eta\tau} \tilde{R}^\eta_\beta \tilde{R}^\tau_\gamma \tag{21b}$$

From Eqn. (21), the partial derivative of all powers can be calculated if $\tilde{A}^a_\gamma$ and $\tilde{\Omega}^\mu_{\eta\tau}$ are known. The following is the formula to calculate them

$$\tilde{A}^a_\beta = \delta^a_\beta + T^{a\mu} E_{\mu\beta} + G^a_L U^L_\beta \tag{22}$$

where $T^{a\mu} = g^{a\mu} - G^a_k P^{\mu k}$; $E_{\mu\beta} = C_k N^k_{\mu\beta}$; $G^a_L = P^{ak} Q_{KL}$; $U^L_\beta = N^L_{\beta\mu}\Delta y^\mu$; $P^{ak} = g^{as} N^K_s$; $C_K = Q_{KL} W^L$; $W^L = N^L_\mu \delta y^\mu$ and $B^{MN} Q_{NK} = \delta^M_K$; $B^{KM} = N^K_r g^{rs} N^M_s = N^K_\gamma g^{\gamma\delta} N^M_\delta + g^{K\delta} N^M_\delta + g^{\delta M} N^K_\delta + g^{KM}$.

With the same method $\tilde{\Omega}^a_{\beta\gamma}$ can be gotten. Therefore $\{\Delta y^a\}$ can be expanded into Taylor's series as

$$\Delta y^a = \tilde{R}^a_\beta \Delta\tilde{y}^\beta + \frac{1}{2}\tilde{\Lambda}^a_{\beta\gamma} \Delta\tilde{y}^\beta \Delta\tilde{y}^\gamma + \cdots \tag{23}$$

whose linear items have not only the first partial derivative but also the second partial derivative, and the second power items include not only the second partial derivative but also the third partial derivative. The formula can improve the accuracy of adjusted values.

3.4 Calculation of adjusted values and parameters and evaluation of their accuracy

After the calculation of $\{\Delta y^a\}$, the parameters based on the transformed relation between $\{\Delta x^\beta\}$ and $\{\Delta y^a\}$ can be gotten, that is

$$\Delta x^a = R^a_\beta \Delta y^\beta + \Lambda^a_{\beta\gamma}\Delta y^\beta \Delta y^\gamma/2 + \Theta^a_{\beta\gamma\delta}\Delta y^\beta \Delta y^\gamma \Delta y^\delta/6 + \cdots \quad (24a)$$

The other $n-m$ adjustments can be gotten with the Monge's model surface

$$\Delta y^L = A^L_\beta \Delta x^\beta + \Omega^L_{\beta\gamma}\Delta x^\beta \Delta x^\gamma/2 + \Phi^L_{\beta\gamma\delta}\Delta y^\beta \Delta y^\gamma \Delta y^\delta/6 + \cdots \quad (24b)$$

From Eqn. (24), the parameter values and the adjusted values of the corresponding adjusted point $\overline{P}$ are

$$\overline{x}^a = x^a + \Delta x^a \quad (25a)$$

$$\overline{y}^r = y^r + \Delta y^r \quad (25b)$$

At the meantime the mean square error of unit weight is

$$m_0 = \sqrt{\frac{\delta\widetilde{y}^r g_{rs} \delta\widetilde{y}^s}{n-m}} = \sqrt{\frac{(y^r_Q - \widetilde{y}^s) g_{rs} (y^s_Q - \widetilde{y}^s)}{n-m}} \quad (26)$$

The covariance of adjusted parameters is

$$K^{\alpha\beta} = R^\alpha_r(\hat{x}) g^{rs} R^\beta_s(\hat{x}) \quad (27)$$

The other adjustments can also be calculated with the same method.

4 ANALYSIS ON CASE

There are error equations

$$\left.\begin{aligned} l^1 &= \cos x^1 + v^1 \\ l^2 &= \sin x^1 + v^2 \end{aligned}\right\}$$

where l^1 and l^2 are measurements which are not relative, $l^1 \sim N(2, 1)$ and $l^2 \sim N(2, 1)$; x^1 is the observing parameter whose initial value is $x^1_0 = \pi/3$; v^1 and v^2 are errors. Then the equation of the model face is

$$l^1 = \cos x^1$$
$$l^2 = \sin x^1$$

From the method mentioned above, we can calculate by four steps as following.

Step 1: to calculate the partial derivative of all powers.

$(A^1_1, A^2_1) = (-0.866, 0.5)$,

$R^1_1 = -1.155$, $N^2_1 = 0.577$

$(\Omega^1_{11}, \Omega^2_{11}) = (-0.5, 0.866)$,

$\Lambda^1_{11} = -0.770$, $N^2_{11} = 1.540$

$(\Phi^1_{111}, \Phi^2_{111}) = (-0.866, 0.5)$,

$\Theta^1_{111} = -3.080$, $N^2_{111} = 3.080$

$N^2_2 = 1$, $N^2_{12} = N^2_{21} = N^2_{22} = 0$

Step 2: to calculate $\delta\widetilde{y}^1$.

$B^{22} = 1.333$, $Q_{22} = 0.750$, $C_2 = 1.50$

$\delta\widetilde{y}^1 = 0.866$, $\widetilde{y}^1 = 1.366$

Step 3: to calculate the coefficiences of the pseudo-multinomial.

$\widetilde{A}^1_1 = 2.536$, $\widetilde{R}^1_1 = 0.394$

Step 4: to calculate all adjusted values.

$\Delta y^1 = \widetilde{y}^1 - y^1 = 0.250$, $\widetilde{y}^1 = 0.750$

$\Delta x^1 = \widetilde{x}^1 - x^1 = -0.321$, $\widetilde{x}^1 = 0.726$

$\Delta y^2 = \widetilde{y}^2 - y^2 = -0.202$, $\widetilde{y}^2 = 0.664$

$m_0 = \pm 1.830$

The method is compared with the linear method, the result is listed in Table 1.

Table 1 Comparison between pseudo-multinomial method and linear method

Method	$\widetilde{x}^1$	$\widetilde{y}^1$	$\widetilde{y}^2$	m_0
Linear method	0.415	0.915	0.403	1.931
Pseudo-multinomial method	0.726	0.750	0.664	1.830

[REFERENCES]

[1] Teunnissen P J G. Nonlinear least squares [J]. Manuscripta Geodaetica, 1990, 15: 21-23.

[2] Blaha G and Besette R P. Nonlinear least squares method via an isomorphic geometrical setup [J]. Bulletin Geodesique, 1989, 63: 115-137.

[3] Teunnissen P J G and Knichmeyer E H. Nonlinearity and least squares [J]. CISM Journal ACSGC, 1989, 42(4): 321-330.

[4] Pope A. Two approaches to nonlinear least-squares adjustment [J]. The Canadian Surveyor, 1992, 28(5): 663-669.

[5] YU Tian-qing. Tensor Analysis and its Mathematical Calculation [M], (in Chinese). Wuhan: Huazhong University of Science and Engineering Press, 1989.

[6] OUYANG Guang-zhong. Calculus of Manifold [M], (in Chinese). Shanghai: Shanghai Science and Technology Press, 1988.

[7] YANG Lu, ZHANG Jing-zhong and HOU Xiao-rong. Nonlinear Algebbaic Equation Group and Instument Prove of Theorem [M], (in Chinese). Shanghai: Shanghai Press of Science, Technology and Education, 1996.

[8] GONG Xiu-jun. Geometric Method of Nonlinear Parameter's Adjustment Based on the Expression of Tensor [D]. Taian: Shandong University of Science and Technology, (in Chinese), 1999.

[9] XIANG Jing-tian and SHI Jiu-en. Statistical Method of Nonlinear Data Process in Nonlinear System [M], (in Chinese). Beijing: Science Press, 1997.

[10] XIE He-ping and XUE Xiu-qian. Mathematical Base and Method of Fractal Application [M], (in Chinese). Beijing: Science and Technology Press, 1997.

(Edited by HUANG Jin-song)