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Genetic optimization of reliability design of machine element①

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[Abstract] Canonical genetic algorithms have the defects of prematurity and stagnation when applied in optimization problems. The causes resulting in such phenomena were analyzed and a class of improved genetic algorithm with niche implemented by crossover of similar individuals and ($\mu+\lambda$) selection was proposed. According to the reliability design theory of machine components, the genetic optimization model of jack clutch was obtained. An optimization instance and some results calculated by improved genetic algorithm were presented. The results of emulations and application show that the improved genetic algorithm with the niche technique can achieve the reliable global convergence and stable convergent velocity almost without any additional calculation expense.

[Key words] genetic optimization; niche; mechanism design; reliability design
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1 INTRODUCTION

The reliability design optimization of machine component is based on the foundation of neoteric mathematical probability, mathematical statistics and optimization. It usually deals with high dimensional, nonconvex and nonlinear object functions subject to various stochastic constraint conditions. Traditional mathematical programming methods in solving such complicated optimization problems have two evident weaknesses. One is that the optimization result tends to be a local extreme when the initial solution is not properly chosen. The other is when an object function is not continuous and differentiable, the methods based on gradient information are thoroughly not applicable.

Genetic Algorithms(GAs)[1,2] are a class of random optimization methods simulating organic evolution and collective genetics. GAs have characteristics such as independence of problems model, self-adaptation, implicit parallelism and robustness of solving various problems, and are successfully used in many large scaled complicated problems. But the canonical genetic algorithms(CGAs) have the defects of premature convergence and stagnation in the later stage of searching.

Empirical data shows that niche[3~5] technique is one of the efficient measures for GAs to avoid getting stuck on local extreme. But the existing implements of niches are at the expenses of reducing the convergent rate of GAs for much additional computation. To alleviate such symptoms, this paper firstly introduced a class of niche technique implemented by crossover of similar individuals and ($\mu+\lambda$) selection into genetic algorithms, which can efficiently improve the premature convergence and stagnation of GAs almost with no additional computation. The results of simulations and optimizing the reliability design of a jack clutch show that the improved genetic algorithm with niche has reliable global convergence and stable convergent velocity.

2 SURVEY OF OPERATIONAL MECHANISM OF CANONICAL GENETIC ALGORITHMS

Canonical genetic algorithms proposed by Holland[1] are usually used to search

$$\max\{f(b) \mid b \in IB^l = \{0, 1\}^l\} \quad (1)$$

where $0 < f(b) < \infty$ and $f(b) \neq$ constant.

CGAs encode each feasible solution b in the solution space of problem (1) as a binary string of length l bits. Each string is said to be an individual, in which the binary bits are called genes. The collection of n individuals is named as a population. The quality of each individual b is evaluated by the fitness function $f(b)$. the one maximizing $f(b)$ is called the global optimum. A canonical genetic algorithm operates individuals in the populations by crossover, mutation and proportional reproduction to search the global optimum until a stopping criterion is fulfilled.

CGAs tend to be premature convergent and stagnant in the later stage of search process. The probabilistic selection mechanism by fitness proportion of individuals makes one with higher fitness possesses more survival probability, but when the fitness of some one is greatly higher than the average over the whole population, the copies of the individual will increase rapidly, even dominate the population, accord-

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ingly resulting in the so called premature convergence. When the search process proceeds to the later stage, most individuals have the fitness approximately equal to the average of the population, the obviously inadequate selection pressure cannot push the evolution of the population, consequently leads to stagnation. Schema theorem[1] provided the theoretical basis to analyze the premature convergence and stagnation of CGAs. Moreover, the random crossover and mutation are likely to destroy the best individual generated over time, and the sampling error of proportional reproduction may probably lose the best one, so that CGAs cannot converge to the global optimum. Rudolph[6] has proved that CGAs are not globally convergent.

The genes recombinations of parental individuals selected randomly can generate new different descendants and increase the probability to explore new search space, but reduce the searching efficiency for completely lacks of direction hints steering the population to evolve. WANG[7] has obtained the result that panmixis leads to the increase of entropy of a population by cybernetics.

Theoretical analysis[8] and empirical data show that the diversity of individuals in populations and selection pressure are two most important factors affecting the searching performance of GAs. Sufficient diversity can improve the global convergence, and higher selection pressure is favorable to speed up the convergent velocity. However, to maintain diversity and increase selection pressure are contradict. Higher selection pressure will lead to lose diversity. Therefore, to pursue the balance between diversity and selection pressure and introduce some heuristic hints to reducing the uncertainty of crossover are the key measures of improving the performance of GAs.

3 NICHE BASED ON SIMILAR STRUCTURE AND ($\mu+\lambda$) SELECTION

Bäck[8] has ever compared the selection pressure of different selection mechanisms by researching the takeover time. The results reveal that ($\mu+\lambda$) selection has the highest selection pressure of the popular mechanisms, which allows the μ parental individual and λ descendants to contest together, ensuring μ individuals with higher fitness to be selected as the members of next generation. Obviously, ($\mu+\lambda$) mechanism implies the global convergence of GAs for saving the best individual generated over time. Emulations also show that ($\mu+\lambda$) mechanism has the quickest rate converging to local extremes. But the fact that all individuals in the population are the same as the local extreme demonstrates the diversity of individuals has decreased to the lowest extent. Therefore, increasing the diversity while applying ($\mu+\lambda$) mechanism to provide higher selection pressure becomes the main goal to improve the design of GAs.

Enlightened by the concept of niche in biology, we change the crossover of randomly selected two parental individuals with probability p_c in the whole population into recombining the genes of two definitely selected individuals with similar fitness, and then, select two better with higher fitness from the parents and two descendants, i. e., after each crossover of two individuals with similar structure, (2+ 2) selection in smaller populations is applied. Mutation is still operated with probability p_m, but (1+ 1) selection is carried out only after the mutation operation over the best one in the population.

The brief description of the improved genetic algorithms with niche implemented by crossover of individuals with similar structure and ($\mu+\lambda$) selection is presented as follows:

```
Initialize population randomly with size 2n;
Evaluate fitness of each individual;
While( stop criterion is not fulfilled)
   Order the population by fitness;
   Construct n pairs of parents sequentially;
   Genes recombination of each parent and
   (2+ 2) selection;
   Mutation with probability p_m, if the mutated
   one is the best then (1+ 1) selection;
   Evaluate fitness of each individual;
Endwhile
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The improved genetic algorithm with the niche technique implies the elitist reservation, it can be proved to be globally convergent by Markov chain of the best individual[9]. The crossover operations of n pairs of parent with similar fitness increase the probability searching independently the global optimum in the disjunctive subspaces, and decrease the uncertainty of descendants. The interaction of crossover and (2 + 2) selection not only provide higher selection pressure but also reduce the lose extent of chromosome diversity.

Using the improved genetic algorithm with niche to maximize the function

$$f_1(x) = \sum_{i=1}^{10} \frac{1}{(b_i(x - a_i))^2 + c_i}$$

over the interval $0 \leqslant x \leqslant 10$ and minimize

$$f_2(x) = -20\exp\left(-0.2\sqrt{\frac{1}{10}\sum_{i=1}^{10} x_i^2}\right) - \exp\left(\frac{1}{10}\sum_{i=1}^{10}\cos(2\pi x_i)\right) + 20 + e$$

over $-20 \leqslant x_i \leqslant 30$, the performance evidently excels that of CGAs.

4 GENETIC OPTIMIZATION OF JACK CLUTCH

4.1 Synopsis of design theory

The structure of a jack clutch is shown in Fig. 1.

Applying optimization methods of reliability design to minimize the bending stress s_b.

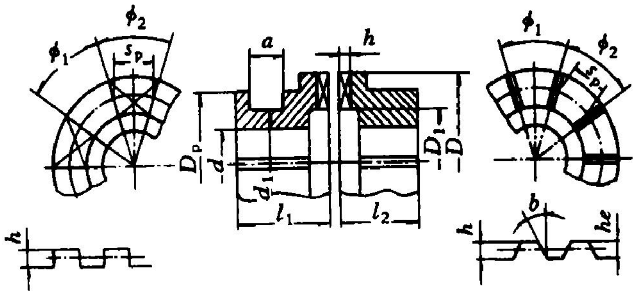

Fig. 1 Structure of jack clutch

According to the design theory[10] of jack clutches, the mean bending stress of a root of tooth is

$$\bar{s}_b = \frac{15.6\bar{T}_n}{\left[\frac{Z}{2}+0.5\right]\left[\frac{\pi\bar{D}(1-\bar{K}_2)}{4Z}\right]^2(1+\bar{K}_1)\bar{D}} \qquad (2)$$

where

$\bar{s}_b$: bending stress of a root of tooth, MPa;

$\bar{T}_n$: torque passes by the jack clutch, N•mm;

Z: the number of teeth;

$\bar{D}$: outer diameter, mm;

$\bar{D}_1$: internal diameter, mm;

$\bar{K}_1 = \bar{D}_1/\bar{D}$: the first structure coefficient;

$\bar{K}_1 = d/\bar{D}$: the second structure coefficient.

Let object vector

$\boldsymbol{X} = (x_1, x_2, x_3, x_4)^T = (Z, \bar{K}_1, \bar{K}_2, \bar{D})^T$

The optimizing task of reliability design is to search

$$\min\{g(\boldsymbol{X}) = \bar{s}_b\} \qquad (3)$$

where the object vector $\boldsymbol{X}$ is subject to a group of constraint conditions.

1) The stochastic constraint condition that the extrusion stress of flank of tooth is not greater than the limit of material tensile stress is

$$P(h_1(\boldsymbol{X}) \leqslant 0) = P(\bar{s}_q \bar{q}_p \leqslant 0) \geqslant a \qquad (4)$$

where

$$\bar{s}_q = \frac{20.8\bar{T}_n}{0.5(1-\bar{K}_1)^2(1+\bar{K}_1)(1+Z)\bar{D}^3} \qquad (5)$$

a is the probability that stochastic constraint condition is fulfilled, q_p is the averaged tensile stress of the material and $\bar{q}_p$ can be chosen to be 1.5[q_p] MPa from engineering experiences.

2) The variables $\bar{K}_1$, $\bar{K}_2$ and $\bar{D}$ should fulfill a set of design specifications as follows:

$$P(\bar{K}_{1\min} \leqslant \bar{K}_1 \leqslant \bar{K}_{1\max}) \geqslant a \qquad (6)$$

$$P(\bar{K}_{2\min} \leqslant \bar{K}_2 \leqslant \bar{K}_{2\max}) \geqslant a \qquad (7)$$

$$P(\bar{D}_{\min} \leqslant \bar{D} \leqslant \bar{D}_{\max}) \geqslant a \qquad (8)$$

$$Z_{\min} \leqslant Z \leqslant Z_{\max} \qquad (9)$$

4.2 Instance of genetic optimization

Assuming that the object random variables and the primary parameters obey normal distribution. Given $T_n = (\bar{T}_n, \sigma_{T_n}) = (12\times 10^4, 0.12\times 10^5)$ N•mm, the allowable tensile stress of material $[q_p] = (60, 6)$ MPa, the outer diameter of clutch $D = (\bar{D}, \sigma_D) = (\bar{D}, 0.001\bar{D})$, the internal diameter $D_1 = (\bar{D}_1, \sigma_{D_1}) = (\bar{D}_1, 0.001\bar{D}_1)$, the caliber $d = (\bar{d}, \sigma_d) = (\bar{d}, 0.0001\bar{d})$. According to engineering experience, (Z, K_1, K_2, D) can be selected respectively between the intervals (25~ 35, 0.7~ 0.75, 1/3~ 1/2, 32~ 65). Design a scheme minimizing the bending stress and subject to that the probability fulfilling all the stochastic constraint conditions is greater than 0.99865.

From the reliability design theory of jack clutches, the constraint Eqns. (4), (6) ~ (9) can be converted into the following definite conditions:

$$h_1(\boldsymbol{X}) = 90 - \bar{s}_q - 3\left[(0.0011\bar{s}_q)^2 + 31.237\right]^{1/2} \geqslant 0 \qquad (10)$$

$$0.7032 \leqslant \bar{K}_1 \leqslant 0.7466 \qquad (11)$$

$$0.3344 \leqslant \bar{K}_2 \leqslant 0.4985 \qquad (12)$$

$$32.0963 \leqslant \bar{D} \leqslant 64.8055 \qquad (13)$$

$$25 \leqslant Z \leqslant 35 \qquad (14)$$

Therefore, according to Eqns. (3) and (10) the penalty function can be constructed as

$$\varphi(\boldsymbol{X}) = g(\boldsymbol{X}) + r^{(t)}(\min(0, h_1(\boldsymbol{X})))^2 \qquad (15)$$

where r is the penalty factor and $t \in IN$ indicates the generations of genetic operation. It is unnecessary to scale the fitness as the improved genetic algorithms with niche employs the $(\mu + \lambda)$ selection. Let

$\boldsymbol{U} = (u_1, u_2, u_3, u_4)$
$= (25, 0.7032, 0.3344, 32.0963)$

$\boldsymbol{V} = (v_1, v_2, v_3, v_4)$
$= (35, 0.7466, 0.4985, 64.8055)$

The minimization of penalty function as Eqn. (15) can be directly converted as the maximization problem to search

$$\max\{f(\boldsymbol{X}) = -\varphi(\boldsymbol{X}) \mid \boldsymbol{U} \leqslant \boldsymbol{X} \leqslant \boldsymbol{V}\} \qquad (16)$$

Encode each solution of the problem (16) as a binary string b of length $4l$, obviously, $b = (b_{4l-1}, b_{4l-2}, \cdots, b_1, b_0) \in \{0,1\}^{4l}$. In order to map each binary string b to a object vector $\boldsymbol{X} \in \Pi_{i=1}^4[u_i, v_i]$, each b will be divided into 4 segments $b_i = (b_{i(l-1)}, b_{i(l-2)}, \cdots, b_{i1}, b_{i0}) \in \{0,1\}^l$ of length l to encode 4 components x_i of an object vector $\boldsymbol{X}$ respectively. So that x_i can be calculated by

$$x_i = u_i + \frac{v_i - u_i}{2^l - 1}\left(\sum_{j=0}^{l-1} b_{ij}\cdot 2^j\right)$$

where changing the value of l can fulfill the arbitrary precision requirement of x_i.

Let $r^0 = 1.009$, $r^{(t)} = r_0^t$, $l = 32$, the population size $n = 100$, mutation probability $p_m = 0.002$, employing the crossover of two points, using repeat number of genetic operations as the stopping criterion. The arithmetic mean minimum values of the object function and the best variables searched are listed in Table 1, where the seeds of random number generator are changed for each employment of the genetic algorithm of 10 times in total.

Table 1 Arithmetic mean of results searched

Repeats	Z	$\overline{K_1}$	$\overline{K_2}$	$\overline{D}$/mm	$g(\boldsymbol{X})$/MPa
Initial	26.8589	0.7174	0.3647	62.8447	696.7032
100	25.0422	0.7442	0.3348	64.7770	692.9336
200	25.0009	0.7465	0.3344	64.8055	692.8029
300	25.0001	0.7466	0.3344	64.8055	692.7990
400	25.0000	0.7466	0.3344	64.8055	692.7988
500	25.0000	0.7466	0.3344	64.8055	692.7988

The data listed in Table 1 show that the improved genetic algorithm can find the global optimum of problem (3) after about 400 repeats in average. As listed in Table 1, the quality of the solution $g(\boldsymbol{X}) = 692.799$ searched by the genetic algorithm with the niche evidently excels that of the following[10]

$$\boldsymbol{X} = (x_1, x_2, x_3, x_4) = (25, 0.74, 0.34, 45)$$

$$g(\boldsymbol{X}) = 2099.264$$

which are obtained by constrained random direction method.

5 CONCLUSIONS

The improved genetic algorithm has obtained stable and better global searching performance by the niche based on the crossover of individuals with similar fitness and $(\mu + \lambda)$ selection, almost without requirement for any additional calculating expense. The genetic optimizing results of the jack clutch are better than those obtained by constrained random direction method. The arbitrary precisions can be obtained by changing the length of binary strings and designing proper stopping criterions.

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