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Calculation of power consumption and induced heat for EMC aluminum ingots①

CAO Zhi-qiang(曹志强), ZHANG Xing-guo(张兴国), LI Zhao-xia(李朝霞), JIN Jun-ze(金俊泽)
(Research Center of Foundry Engineering, Dalian University of Technology, Dalian 116024, P. R. China)

[Abstract] The electrical parameters and power consumption in electromagnetic casting of aluminum ingots were calculated and discussed in detail. Moreover, the induced heat was calculated with the eddy current within the liquid column. It is found that the calculated values agree with the measured results. Once the inductor current was given, the magnetic flux density in electromagnetic casting could be calculated and the electromagnetic pressure could be obtained. The key to the EMC is the balance between the electromagnetic pressure and the metallostatic pressure. As the liquid column, controlled by the casting speed and pouring speed through a magnetic sensor, is kept away from the inductor, a gap forms linear relationship between the inductor and ingot. The bigger the current is, the smaller the ingot size is.

[Key words] electromagnetic casting; electromagnetic pressure; induced heat; power consumption
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1 INTRODUCTION

Electromagnetic casting (EMC) is a kind of continuous casting technique without mould[1~5]. It has been widely used in Europe and America and also is being paid great attention in China recently[6~8], because its ingots have smooth surface, homogeneous microstructure and good hot workability.

Unlike direct chill casting (DCC), the liquid column of aluminum ingots in EMC depends on the support of electromagnetic forces acting on the periphery of it and doesn't contact with the inductor.

As shown in Fig. 1, the inductor is fed with an alternating current at medium frequency, generating an alternating electromagnetic field. This field, in turn, creates eddy currents in the surface layers of the liquid metal, which interact with the magnetic field to provide a force directed inwards towards the center of the liquid metal pool. This force counteracts the metallostatic head of the column of liquid metal, and make the liquid column conform to the contour of the inductor without touching it and be cooled and solidified by continuous water impingement.

Except the process parameters, the electric parameters such as current, voltage, and power consumption should be grasped for the successful operation of EMC.

In this paper, an equivalent mutual inductance model is developed to calculate the electrical parameters and power consumption to design the inductor and optimize the processing parameters.

Early researchers only paid attention to the axial distribution of electromagnetic pressure. No report on the lateral electromagnetic induction was found. Hereinafter a mathematical model is supposed in the first time to calculate the relationship between current and ingot size. In addition, the induced heat is also calculated and verified by the experimental data.

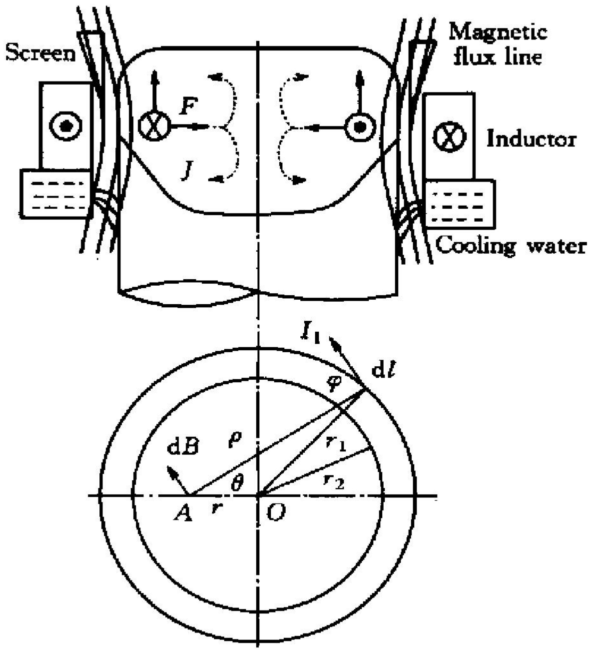

Fig. 1 Schematic illustration of electromagnetic casting

2 CALCULATION OF ELECTRIC PARAMETERS

2.1 Current and ingot size

As shown in Fig. 1, the expression of magnetic field caused by inductor current unit $I_1 dl$, at point A

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of cross section is

$$dB = \mu I_1 dl \sin\varphi/(4\pi\rho^2) \quad (1)$$

where μ is the magnetic permeability, I_1 is the effective current in the inductor, dl is the unit length of conductor, dB is the magnetic flux density induced by current unit at point A, and ρ is the distance from point A to the current unit.

$$\because dl \cdot \sin\varphi = \rho d\theta \quad (2)$$

$$\therefore dB = \mu I_1 d\theta/(4\pi\rho) \quad (3)$$

From the law of cosines, we know

$$\rho^2 + r^2 - 2r\rho\cos\theta = r_1^2 \quad (4)$$

Substituting Eqn. (4) into Eqn. (3), there is a relationship between magnetic flux density of point A and the inductor current unit $I_1 dl$

$$dB = \frac{\mu I_1}{4\pi r_1} \cdot \frac{d\theta}{\frac{r}{r_1}\cos\theta + \sqrt{1 - \frac{r^2}{r_1^2}\sin^2\theta}} = -\frac{\mu I_1 r_1}{4\pi(r_1^2 - r^2)} \cdot \left[\frac{r}{r_1}\cos\theta - \sqrt{1 - \frac{r^2}{r_1^2}\sin^2\theta}\right] d\theta \quad (5)$$

where r_1 is the calculation radius of inductor, $r_1 = r_{01} + \delta_1/2$, r_{01} is the radius of inductor, δ_1 is the skin depth of copper inductor.

Integrating Eqn. (5), the magnetic flux density of point A induced by the current I_1 becomes

$$B = 2\int_0^{\pi} dB = \frac{\mu I_1}{2r_1(1-\beta^2)} \cdot \left\{1 - \left(\frac{1}{2}\right)^2\beta^2 - \frac{1}{3}\left(\frac{1\cdot 3}{2\cdot 4}\right)^2\beta^4 - \dots - \left[\frac{(2n-1)!!}{2^n n!}\right]^2 \frac{\beta^{2n}}{2n-1}\right\} \quad (6)$$

where $\beta = r_2/r_1$, $r_2 = (1+\alpha)r_{02} - \delta_2/2$, δ_2 and α are the skin depth and linear contraction rate of liquid aluminum respectively, r_{02} is the radius of ingot, r_2 is the radius of liquid column.

When the pressure due to surface tension is ignored, the relationship between the magnetic flux density and the height of liquid column is[9, 10]

$$\rho g h_z = p_E = B^2/(2\mu) = \mu H^2/2 \quad (7)$$

Substituting Eqn. (6) into Eqn. (7), the relationship between the current and the size of the ingot for a certain inductor size can be got:

$$I_1 = \frac{2r_1 B f(\beta)}{\mu} \quad (8)$$

$$f(\beta) = (1-\beta^2)/\left\{1 - \left(\frac{1}{2}\right)^2\beta^2 - \frac{1}{3}\left(\frac{1\cdot 3}{2\cdot 4}\right)^2\beta^4 - \dots - \left[\frac{(2n-1)!!}{2^n n!}\right]^2 \frac{\beta^{2n}}{2n-1}\right\} \quad (9)$$

Fig. 2 shows the relationship of the current and the size of the ingot for d0.21 m inductor. The bigger the current is, the smaller the ingot size is, i. e., the farther the distance between the ingot and the inductor is. It should be noted that the calculated current refers to the Kaiser type inductor of no screen.

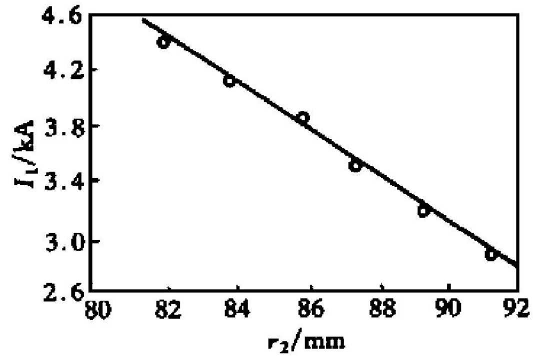

Fig. 2 Electrical intensity (I_1) versus ingot radius (r_2)

2.2 Impedance and power consumption

The equivalent circuit of inductor and ingot system is shown in Fig. 3. From the theory about hollow transformer, we can write[10]

$$(R + jX_1)\dot{I} - jX_m\dot{I}_1 = 0 \quad (10)$$

$$\dot{I}_1 \approx \dot{I}_d \quad (11)$$

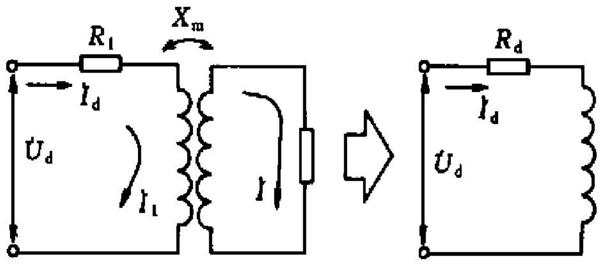

Fig. 3 Equivalent circuit of inductor-ingot system
R_1, R, R_d—Resistance of coil, ingot and the equivalent;
X_1, X, X_d—Reactance of coil, ingot and the equivalent;
X_m—Mutual inductive reactance of inductor and ingot

Combining these equations and solve them, we get

$$\dot{U}_d = \left(R_1 + \frac{X_m^2}{R^2 + X^2}R\right) + j\left(X_1 - \frac{X_m^2}{R^2 + X^2}X\right)\dot{I}_d \quad (12)$$

Let

$$k = \frac{X_m^2}{X^2 + R^2} \quad (13)$$

There is

$$R_d = R_1 + kR \quad (14)$$

$$X_d = X_1 - kX \quad (15)$$

$$Z_d = \sqrt{R_d^2 + X_d^2} \quad (16)$$

The eddy current usually drops with exponential function. That is to say, the current is concentrated in the skin depth. For the convenience of calculation, we regard the ingot as a tube with the thickness of skin depth and assume the current is approximately uniform in the skin depth[10]. Then the electrical resistance of ingot is

$$R = \rho L/S = 2\rho(a + b - 2\delta)/(h_z\delta) \quad (17)$$

The electrical reactance of ingot is

$$X = \omega S \mu K_L / h_z = 2\pi f \mu K_L (a - \delta)(b - \delta)/h_z \qquad (18)$$

where $\rho = 26 \times 10^{-8}$ Ω·m, is the electrical resistivity of the liquid aluminum, a and b are the length and width of cross section respectively, K_L is the modified factor of the self-induction and can be reckoned from Fig. 4 according to $\lambda = (D - \delta)/h_z$, here $D = 2\sqrt{ab/\pi}$.

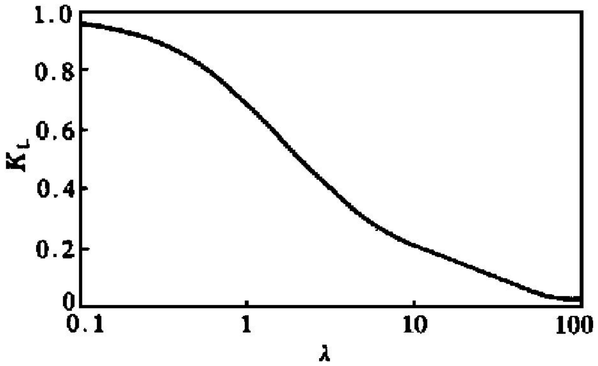

Fig. 4 Modified factor of coil length

Similarly, the electrical resistance of inductor is

$$R_1 = \rho_1 L / S_1 = 2\rho_1 (a_1 + b_1 + 2\delta_1)/(h_1 \delta_1) \qquad (19)$$

The reactance of the inductor is

$$X_1 = \omega S_1 \mu K_{L1} / h_1 = 2\pi f \mu (a_1 + \delta_1)(b_1 + \delta_1) \cdot K_{L1}/h_1 \qquad (20)$$

where $\rho_1 = 2.0 \times 10^{-8}$ Ω·m, is the electrical resistivity of pure copper inductor.

The mutual inductance of the inductor-ingot system is

$$X_m = \omega S \mu K_m / (2h_z) = 2\pi f \mu (a - \delta)(b - \delta) K_m / (2h_z) \qquad (21)$$

where K_m is the modified factor of mutual induction and its value can be estimated from Fig. 5.

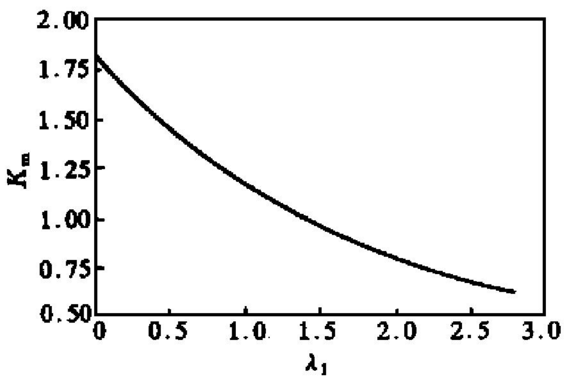

Fig. 5 Modified factor of mutual inductance

After such parameters as R_d, Z_d and X_d have been calculated according to Eqns. (14), (15) and (16), the voltage of the inductor can be written as

$$U_d = I_d \cdot Z_d \qquad (22)$$

The power consumption of the inductor is

$$P_d = I_d^2 \cdot R_d \qquad (23)$$

For the 0.52 m × 0.13 m pure aluminum ingots, the calculated results are shown in Table 1.

Table 1 Calculated electric parameters for EMC pure aluminum ingots

Electric parameters	$R/10^{-3}$ Ω	$R_1/10^{-3}$ Ω	$X/10^{-3}$ Ω
Calculated values	1.67	4.96	8.39
Electric parameters	$X_1/10^{-3}$ Ω	$X_m/10^{-3}$ Ω	k
Calculated values	9.92	5.29	0.38
Electric parameters	$R_d/10^{-3}$ Ω	$X_d/10^{-3}$ Ω	$Z_d/10^{-3}$ Ω
Calculated values	1.13	6.73	6.82
Electric parameters	U_d/V	P_d/kW	I_1/A
Calculated values	33	27	4 643

It is noted that the voltage is 34 V, the current is 4 643 A, and the power consumption is 27 kW when the height of liquid column is 0.04 m. The calculated results agree well with the practical experimental data.

Table 2 gives the calculated energy consumption per unit length or unit mass for three kinds of EMC ingots. It can be seen that with the increase of ingot size, the power consumption of unit length increases, while the power consumption of unit mass decreases remarkably.

Table 2 Calculated energy consumption of ingots per unit length or unit mass

Ingot sizes	Energy consumption, W_1/(kWh·m^{-1})	Energy consumption W_2/(kWh·kg^{-1})
d0.1 m	3	0.160
0.52 m × 0.13 m	11	0.060
1.28 m × 0.34 m	17	0.014

2.3 Induced heat

According to the Faraday's electromagnetic field theory of alternating current, the electromagnetic force acted on the liquid metal is[11]

$$P_E = \frac{1}{2\pi R h} \iiint J \cdot B \cos\varphi \, \mathrm{d}V = \frac{1}{R}\int_0^R J \cdot B \cos\varphi \cdot r \, \mathrm{d}r = B_0^2/(2\mu) \qquad (24)$$

Noting that both the current density and the magnetic flux density distribute as exponential function due to the skin effect

$$J = J_0 \cdot \exp[(r - R)/\delta] \qquad (25)$$

$$B = B_0 \cdot \exp[(r - R)/\delta] \qquad (26)$$

Substituting Eqns. (25) and (26) into Eqn. (24), then

$$J_0 = B_0/(\mu\delta\cos\varphi) = \frac{2}{\mu\delta} \cdot \sqrt{\mu\rho g h} \qquad (27)$$

Here $B_0 = \sqrt{2\mu\rho g h}$, $\varphi = 45°$ a phase difference between the current density and the transverse magnetic field penetrating into a plane[11]. Thus the eddy current intensity is

$$I = \iint J \mathrm{d}s = h\int_0^R J_0 \cdot \exp[(r - R)/\delta] \mathrm{d}r = \frac{2}{3} h\delta J_0 \quad (28)$$

The induced heat power is

$$P = \iiint \frac{J^2}{\sigma} \mathrm{d}V = \frac{1}{4\sigma} J_0^2 \cdot h\delta\pi R \quad (29)$$

According to Eqn. (29), the temperature increment of the skin depth per unit time is

$$\frac{\Delta T}{\tau} = P/(c\rho 2\pi R\delta h) \quad (30)$$

where $c = 899$ J/(kg·K), $\rho = 2\,700$ kg/m^3.

The actual measured temperature increment of d0.2 m ingot is 3 K/s, which was obtained by five concentric aluminum circles of 1.9 mm thickness and 10 mm height[12]. The measured value is lower than the theoretical result of 3.5 K/s because of the heat exchange loss of air convection.

Table 3 shows the calculated results of induced heat, induced current, electromagnetic pressure and temperature increment, when the current is 4 300 A, the liquid column height is 0.045 m.

Table 3 Calculated induced heat etc for d0.164 m ingot

Parameters	P/W	p_E/(N·m^{-2})	$\frac{\Delta T}{\tau}$/(K·s^{-1})
Calculated values	1 000	1 058	3.5
Parameters	I/A	J_0/(A·m^{-2})	
Calculated values	1 740	1.16×10^7	

3 CONCLUSIONS

1) A linear relationship between the inductor current and ingot size was found.

2) The calculated induced heat agrees well with the measured one.

3) The electrical parameters and power consumption used in electromagnetic casting of aluminum ingots were calculated and discussed in detail.

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