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Unsteady lubrication modeling of inlet zone in metal rolling processes①

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[Abstract] An unsteady lubrication model of inlet zone in metal rolling was established. The simulation computations show that for the variation amplitude of the inlet film thickness, the variation of the inlet angle contributes the largest, the surface mean speed contributes the second and the back tension stress the least. The higher the input frequency is, the smaller the amplitude output of the inlet film thickness will be. For a sinusoidal input, the inlet film thickness varies periodically but is not a sine wave because the system is not linear.

[Key words] metal rolling; unsteady state lubrication; inlet film thickness
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1 INTRODUCTION

Metal rolling process usually runs in the steady state. When the mill structure is undergoing a self-excited vibration known as chatter, the lubricant flow between the roll and the strip surface is no longer in the steady state. Many experimental investigations have pointed out that lubrication is one of the main factors causing chatter. But very few of the investigators have made theoretical analysis to explain how and why chatter occurs through unsteady lubrication phenomenon. Moller and Hoggart[1] concluded that torsional vibration originated from a sudden change in torque, but the vibration was shown to be stable and self-sustaining only when the coefficient of friction decreases with increasing speed. They also reported that the rolling speed and the concentration of paraffin in kerosene would change the torque variation amplitude, which was believed to be associated with lubrication. Yarita et al[2] concluded from their experimental results that chatter in the cold rolling process is related to instability of the emulation and fluctuations of the tension in the strip. They also suggested that the use of a lubricant with good lubricity successfully prevent chatter. Tamiya, Furui and Iida[3] showed that chatter is a self-vibration caused by the phase difference in tension when the influence of change in tension on the strip thickness is large. Gallenstein[4] established the range of rolling conditions over which torsional chatter could be produced. He found that chatter would occur in a certain range of total roll force and mill speed. When the mill speed was varied, the chatter amplitude was also changed.

All existing models[5~11] of friction in rolling treat the steady state and are not able to capture the variations in friction under the rapidly changing conditions occurring during chatter. When the process becomes time dependent, due to roll mill vibration or chatter, the unsteady term in Reynolds equation is no longer negligible. Vibrations in various parts of the mill structure may induce a significant impact on hydrodynamic lubrication. Too large a hydrodynamic effect may cause very high film thickness resulting in poor surface quality, while too small a hydrodynamic effect may induce severe metal-to-metal contact and consequent galling and pickup. Alternating high and low film thickness regions may lead to a "stripped" surface that could result in the sheet being scrapped.

2 THEORETICAL MODELING

When the rolling speed is high, or the lubricant viscosity is large, the metal rolling process operates in the thick film lubrication regime. Fig. 1 shows the process geometry. Fig. 2 is the geometry of inlet zone. It is assumed that there are no surface roughness or thermal effects involved.

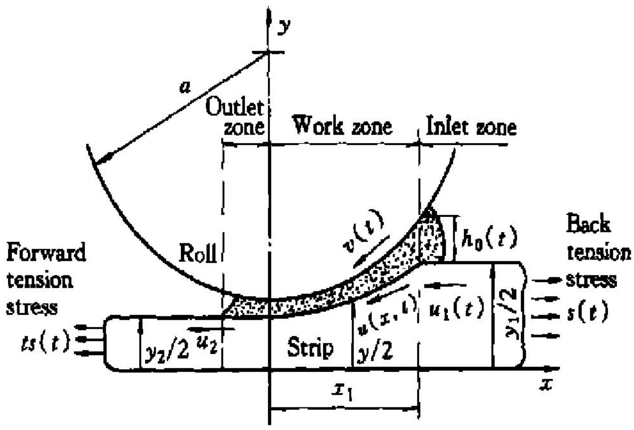

Fig. 1 Geometry of roll bite

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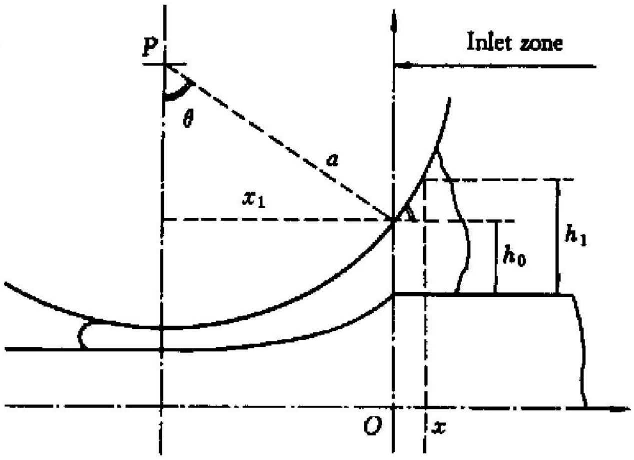

Fig. 2 Geometry of inlet zone

As the first step to understand the unsteady lubrication mechanism in strip rolling, an inlet analysis will be developed. The key result from this analysis will be the inlet film thickness. Unsteady variables, such as tensions, rolling speed, inlet angle, will be taken into account. The variation of the inlet film thickness is very important because it indicates the lubricant quantity that is being carried into roll bite.

In the inlet zone, the strip is assumed to be rigid. The separation between the strip and the roll is caused by hydrodynamic pressure that is built up by wedge and squeeze action. The pressure p generated in the inlet zone is given by the Reynolds equation

$$\frac{\partial}{\partial x}\left[\frac{h_1^3}{12\mu}\frac{\partial p}{\partial x}\right] = -\bar{u}_1\frac{\partial h_1}{\partial x} + \frac{\partial h_1}{\partial t} \qquad (1)$$

where h_1 is the lubricant film thickness, μ is the lubricant viscosity, t is the time, $\bar{u}_1$ is the mean surface speed of the strip and the roll.

In the inlet zone the geometry can be described as a function of position x:

$$h_1 = h_0 + \theta x \qquad (2)$$

where h_0 is the film thickness at the inlet edge of the work zone referred to as the inlet film thickness. θ is the inlet angle that is defined as the scope $\partial h_1/\partial x$ at the inlet edge of the work zone. Since all variables are time dependent due to the vibration of the mil, the inlet zone geometry varies with time. The squeeze term on the right hand side of Eq. (1) can be determined as

$$\frac{\partial h_1}{\partial t} = \dot{h}_0 + x\dot{\theta} \qquad (3)$$

Substituting Eqn. (3) into Eqn. (1) and integrating it with respect to x yields

$$\frac{h_1^3}{12\mu}\frac{\partial p}{\partial x} = (\dot{h}_0 - \theta\bar{u}_1)x + \frac{\dot{\theta}}{2}x^2 + f(t) \qquad (4)$$

where $f(t)$ is an arbitrary function of time. A negligible pressure gradient is assumed at the inlet edge of the work zone (where $h_1 = h_0$) so that $f(t)$ is zero.

Since the pressure in the roll bite is large, the viscosity μ is assumed to vary with pressure according to the Barus equation

$$\mu = \mu_0 e^{\gamma p} \qquad (5)$$

where μ_0 is the viscosity at atmospheric pressure and γ the pressure coefficient of viscosity. To simplify the analysis, another pressure variable Φ may be defined by

$$\Phi = e^{-\gamma p} \qquad (6)$$

So that Eqn. (4) becomes

$$\frac{\partial\Phi}{\partial x} = 6\gamma\mu_0\left[2\theta\bar{u}_1\frac{x}{h_1^3} - 2\dot{h}_0\frac{x}{h_1^3} - \dot{\theta}\frac{x^2}{h_1^3}\right] \qquad (7)$$

Eqn. (7) implies that there are three terms that contribute to the pressure variation along the rolling direction. The first term on the right hand side of Eqn. (7), i. e.

$$\Phi_1' = 12\gamma\mu_0\theta\bar{u}_1\frac{x}{h_1^3} \qquad (8)$$

portrays the wedge effect in which the lubricant is carried by the converging moving surfaces. When the vibrating roll moves downward and bites deeper into the strip, the inlet edge moves outward and the inlet angle becomes larger. If the inlet angle varies significantly due to the position shift of the inlet edge, this may dramatically change the inlet film thickness. The second term

$$\Phi_2' = -12\gamma\mu_0\dot{h}_0\frac{x}{h_1^3} \qquad (9)$$

and the third term

$$\Phi_3' = -6\gamma\mu_0\dot{\theta}\frac{x^2}{h_1^3} \qquad (10)$$

capture squeeze action by up-and-down roll motion. The second term (direct squeeze term) depicts the influence of inlet film thickness variation while the third term (tilt squeeze term) represents the influence of the inlet angle variation rate $\dot{\theta}$. The higher the frequency of the roll vibration, the faster the inlet angle varies. That is, a higher frequency unsteady motion makes these terms more important.

To simplify the derivation, two different geometric approximations will be used. For the first two terms of Eqns. (8) and (9), the linear approximation from Eqn. (2) will be used. However, due to a problem with the infinity boundary condition when applying the linear approximation to the $\dot{\theta}$ term, a parabolic approximation

$$h_1 = h_0 + \frac{x_1}{a}x + \frac{x^2}{2a} \doteq h_0 + \theta x + \frac{x^2}{2a} \qquad (11)$$

is adopted in treating the tilt squeeze term, where a is the roll radius.

It is easier to integrate Eqn. (7) if it is separated into two parts. The first part combines the wedge term with the direct squeeze term

$$\frac{\partial\Phi_a}{\partial x} = \frac{12\gamma\mu_0 x}{h_1^3}(\theta\bar{u}_1 - \dot{h}_0) \qquad (12)$$

which can be rewritten in terms of h_1 by substituting for x from Eqn. (2)

$$\frac{\partial \phi_a}{\partial h_1} = \frac{12\gamma\mu_0}{h_1^3\theta}\left[\bar{u}_1 - \frac{h_0}{\theta}\right](h_1 - h_0) \tag{13}$$

Integrating Eqn. (13) with respect to h_1 yields

$$\phi_a = \frac{12\gamma\mu_0}{\theta}\left[\bar{u}_1 - \frac{h_0}{\theta}\right]\left[-\frac{1}{h_1} + \frac{h_0}{2h_1^2}\right] + f_a(t) \tag{14}$$

The second part, which contains only the tilt term Eqn. (10), is written as

$$\frac{\partial \phi_b}{\partial x} = -6\gamma\mu_0\dot{\theta}\frac{x^2}{h_1^3} \tag{15}$$

Substituting for h_1 from Eqn. (11) and integrating in terms of x yields

$$\phi_b = -6\gamma\mu_0\dot{\theta}\cdot\left\{-\frac{4a^3[(a\theta^2 - 2h_0)x + h_0(x + a\theta)]}{(a\theta^2 - 2h_0)(2ah_0 + 2a\theta x + x^2)^2} + \frac{2a^2(h_0 + a\theta^2)(x + a\theta)}{(a\theta^2 - 2h_0)^2(2ah_0 + 2a\theta x + x^2)} + \frac{a^2(h_0 + a\theta^2)}{(a\theta^2 - 2h_0)^2\sqrt{a(a\theta^2 - 2h_0)}}\cdot \ln\left|\frac{x + a\theta - \sqrt{a(a\theta^2 - 2h_0)}}{x + a\theta + \sqrt{a(a\theta^2 - 2h_0)}}\right|\right\} + f_b(t) \tag{16}$$

The complete reduced pressure ϕ now can be obtained by superposition as

$$\phi = \phi_a + \phi_b + f(t) \tag{17}$$

where

$$f(t) = f_a(t) + f_b(t) \tag{18}$$

Since the reduced pressure ϕ tends to unity as x and h_1 tends to infinity, the upstream boundary condition

$$x = \infty,\ h_1 = \infty,\ \text{and}\ \phi = 1 \tag{19}$$

will be applied. After applying the boundary condition, the arbitrary function of time is obtained as

$$f(t) = 1 \tag{20}$$

Therefore, the complete reduced pressure ϕ can be written as

$$\phi = 1 + \frac{12\gamma\mu_0}{\theta}\left[\bar{u}_1 - \frac{h_0}{\theta}\right]\left[-\frac{1}{h_1} + \frac{h_0}{2h_1^2}\right] - 6\gamma\mu_0\dot{\theta}\cdot\left\{-\frac{4a^3[(a\theta^2 - 2h_0)x + h_0(x + a\theta)]}{(a\theta^2 - 2h_0)(2ah_0 + 2a\theta x + x^2)^2} + \frac{2a^2(h_0 + a\theta^2)(x + a\theta)}{(a\theta^2 - 2h_0)^2(2ah_0 + 2a\theta x + x^2)} + \frac{a^2(h_0 + a\theta^2)}{(a\theta^2 - 2h_0)^2\sqrt{a(a\theta^2 - 2h_0)}}\cdot \ln\left|\frac{x + a\theta - \sqrt{a(a\theta^2 - 2h_0)}}{x + a\theta + \sqrt{a(a\theta^2 - 2h_0)}}\right|\right\} \tag{21}$$

At the inlet edge of the work zone, the pressure can be obtained by applying the Tresca yield criterion

$$x = 0,\ h_1 = h_0\ \text{and}\ p = \sigma - s \tag{22}$$

where σ is the material flow stress and s is the back stress. The reduced pressure at this point is

$$\phi = e^{-\gamma(\sigma - s)} \tag{23}$$

Substituting Eqns. (22) and (23) into Eqn. (21) yields

$$1 - e^{-\gamma(\sigma - s)} = \frac{6\gamma\mu_0}{\theta h_0}\left[\bar{u}_1 - \frac{h_0}{\theta}\right] + 6\gamma\mu_0\dot{\theta}C_R \tag{24}$$

where the inlet angle variation rate factor C_R is given by

$$C_R = \frac{3a^2\theta}{(a\theta^2 - 2h_0)^2} + \frac{a^2(h_0 + a\theta^2)}{(a\theta^2 - 2h_0)^2\sqrt{a(a\theta^2 - 2h_0)}}\cdot \ln\left|\frac{a\theta - \sqrt{a(a\theta^2 - 2h_0)}}{a\theta + \sqrt{a(a\theta^2 - 2h_0)}}\right| \tag{25}$$

The inlet film thickness variation rate can also be determined by rearranging Eqn. (24)

$$\dot{h} = \theta\bar{u}_1 - \frac{\theta^2 h_0[1 - e^{-\gamma(\sigma - s)}]}{6\gamma\mu_0} + \theta^2 h_0\dot{\theta}C_R \tag{26}$$

Four basic variables are important to the inlet film thickness variation: back tension s, mean surface speed $\bar{u}_1$, inlet angle θ and its rate of variation $\dot{\theta}$.

The variation of the mean surface speed $\bar{u}_1$ may be from two different sources. One is from the mill speed variation, which is directly from the accelerated mill motions due to the mill torque variation. The other one is from the inlet strip speed variation, which is due to the variations of the friction and the back tension stress s. The back tension stress variation is induced by the relative motion between the inlet edge of the present stand and the outlet edge of the prior stand. The inlet angle θ and the inlet angle variation rate $\dot{\theta}$ are due to the roll translation and velocity relative to the strip.

When the unsteady terms in Eqn. (26) are eliminated, Eqn. (26) becomes

$$h_0 = \frac{6\gamma\mu_0\bar{u}_1}{\theta[1 - e^{-\gamma(\sigma - s)}]} \tag{27}$$

which is the isothermal Wilson-Walowit[12] inlet film thickness with back tension stress. This result is not a surprise because unsteady lubrication is simply an extension of steady state lubrication. The inlet film thickness is changing at a rate determined by the current conditions. Thus the current film thickness reflects the history of the operating conditions. The phenomenon shown at each moment is simply the transported result of the last moment plus the input of the operation conditions at the current moment. It should stay unchanged while there is no structure changes with respect to time.

3 NONDIMENSIONAL INLET FILM THICKNESS

Nondimensionalization of the variables can help to deeply understand the physical meaning of an equation. Here because our concern is to see the influence of the unsteady terms, each unsteady time dependent variable will be divided into an unsteady state value and a steady state one that is known and fixed under certain condition. For example, the back tension stress s can be written as

$$s = s_0 + \delta s \tag{28}$$

where s_0 is the steady component and δs is the variation. Similarly

$$u_1 = u_{10} + \delta u_1,\quad v = v_0 + \delta v,$$

$$\bar{u}_1 = \frac{u_1 + v}{2},\quad \theta = \theta_0 + \delta\theta$$

where u_{10}, v_0, θ_0 are the steady state strip speed, roll speed and inlet angle respectively and δu_1, δv, $\delta\theta$ are the corresponding unsteady components. The latter can be seen as inputs to the unsteady inlet lubrication problem.

The following nondimensional variables can be defined as

$$H_0 = \frac{h_0}{h_{00}},\quad U_1 = \frac{u_1}{v_0},\quad V = \frac{v}{v_0},\quad \Theta = \frac{\theta}{\theta_0}$$

where h_{00} is the steady state inlet film thickness:

$$h_{00} = \frac{6\gamma\mu_0\bar{u}_{10}}{\theta_0[1 - e^{-\gamma(\sigma - s_0)}]} \tag{29}$$

The nondimensional time scale in the inlet zone is defined as

$$T_i = \frac{t}{(h_{00}/v_0\theta_0)} = \frac{tv_0\theta_0}{h_{00}} \tag{30}$$

The denominator $h_{00}/v_0\theta_0$ is roughly the time period for the lubricant to flow through the length of the inlet zone when the lubrication system is in steady state. Using these nondimensional variables defined above, Eqn. (26) becomes

$$\dot{H}_0 = \Theta\bar{U}_1 - \bar{U}_{10}\Theta^2 H_0\frac{1 - e^{-\gamma(\sigma - s)}}{1 - e^{-\gamma(\sigma - s_0)}} + \Theta^2 H_0\dot{\Theta}C_{R1} \tag{31}$$

where $\bar{U}_{10}$ is the nondimensional steady state mean inlet surface speed defined by

$$\bar{U}_{10} = \frac{(u_{10} + v_0)/2}{v_0} = \frac{\bar{u}_{10}}{v_0},$$

$$\bar{U}_1 = \frac{\bar{u}_1}{v_0} \tag{32}$$

$$C_{R1} = \theta_0^3 C_R$$

or

$$C_{R1} = \frac{3\Theta}{(\Theta^2 - 2CH_0)^2} + \frac{\Theta^2 + CH_0}{(\Theta^2 - 2CH_0)^2\sqrt{\Theta^2 - 2CH_0}} \cdot \ln\frac{\Theta - \sqrt{\Theta^2 - 2CH_0}}{\Theta + \sqrt{\Theta^2 - 2CH_0}} \tag{33}$$

where C is a constant given by

$$C = \frac{h_{00}}{a\theta_0^2} \tag{34}$$

A Matlab program using a 4th order Runge-Kutta method has been developed to solve Eqn. (30) for the inlet film thickness H_0. The program is designed to use sinusoidal inputs either independently or combined so that

$$\delta s = \delta s_0 \sin\omega t,\quad \delta u_1 = \delta u_{10}\sin\omega t,$$

$$\delta v = \delta v_0\sin\omega t,\quad \delta\theta = \delta\theta_0\sin\omega t \tag{35}$$

The time derivative of Eqn. (35) is

$$\dot{\theta} = \delta\theta_0\,\omega\cos\omega t \tag{36}$$

The frequency of vibration can also be nondimensionalized as

$$\Omega_i = \frac{\omega h_{00}}{v_0\theta_0} \tag{37}$$

thus

$$\Omega_i T_i = \omega t \tag{38}$$

4 AN EXAMPLE COMPUTATION

The rolling mill and lubricant characteristics are listed in Table 1. Figs. 3, 4, 5 are respectively inlet film thickness variations with sinusoidal back tension stress s, surface mean speed $\bar{u}_1$, inlet angle θ. The inlet film thickness solutions with different back tension stress input frequencies are plotted in Fig. 6. The angular displacement shown here is the product of nondimensional angular frequency and nondimensional

Table 1 Condition for lubricant characteristics

Initial strip thickness, y_{10}/mm	Strip flow stress, σ/MPa	Back tension stress, s/MPa	Reduction, R/mm
0.254	552	138	025
Roll radius, a/mm	**Roll speed, v_0/(m·s^{-1})**	**Lubricant base viscosity, μ_0/(Pa·s)**	**Lubricant pressure coefficient, γ/MPa^{-1}**
76.2	10.2	0.0689	0.0145

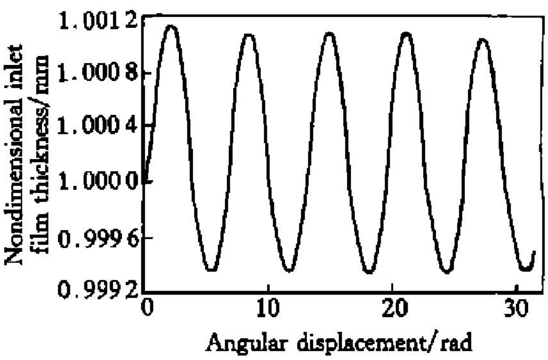

Fig. 3 Inlet film thickness variations with sinusoidal back tension stress
($\Omega_i = 1.0$, $\delta s_0/s_0 = 25\%$)

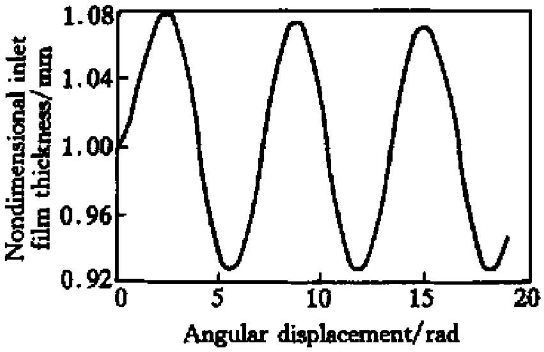

Fig. 4 Inlet film thickness variations with sinusoidal roll and strip speeds
($\Omega_i = 1.0$, $\delta\bar{u}_{10}/\bar{u}_{10} = 10\%$)

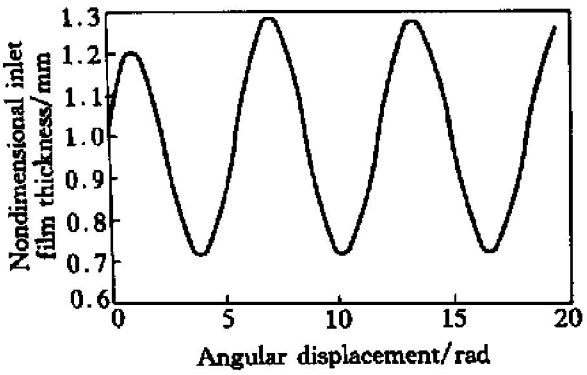

Fig. 5 Inlet film thickness vriations with sinusoidal inlet angle
($\Omega_i = 1.0$, $\delta\theta_0/\theta_0 = 0.35\%$)

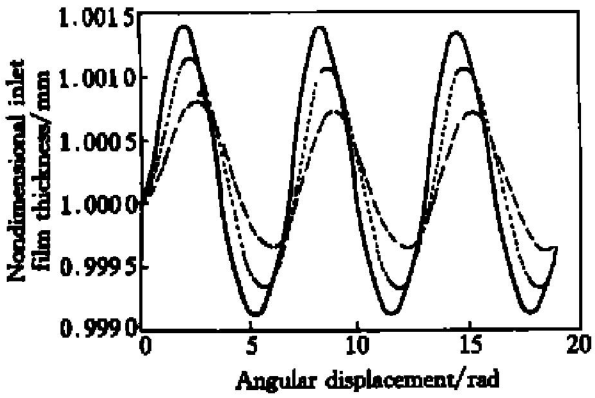

Fig. 6 Inlet film thickness variations with sinusoidal back tension stress of various frequencies
——Nondimensional Freq. = 0.5;
……Nondimensional Freq. = 1.0;
-----Nondimensional Freq. = 2.0

time.

For a nondimensional frequency of 1.0, a sinusoidal back tension stress input with a variation amplitude of 25% results in an output of only 0.1% amplitude (Fig. 3). On the other hand, a sinusoidal roll and strip speed input with only 10% amplitude can obtain an output as large as almost 1% (Fig. 4), while the inlet angle case can reach almost an output of 2% (Fig. 5). The solutions for higher frequency inputs have smaller amplitude outputs. The inlet film thickness varies periodically but is not a sine wave because the system is not linear.

5 CONCLUSION

The unsteady lubricating model established in this paper contains the equation of Wilson and Walowit[12] for the steady state lubrication in strip rolling process. It has important theoretical value and practical sense. The simulation computations show that for the variation amplitude of the inlet film thickness, the variation of the inlet angle contributes the largest, the surface mean speed the second and the back tension stress the least. The solutions for higher frequency inputs have smaller amplitude outputs. The inlet film thickness varies periodically but is not a sine wave because the system is not linear.

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