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Investigation for parametric vibration of rolling mill ①

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[Abstract] The vibration unsteady condition of rolling mill caused by flexural vibration of strip has been investigated. The parametric flexural vibration equation of rolled strip has been established. The parametric flexural vibration stability of rolled strip has been studied and the regions of stability and unstability have been determined based on Floquet theory and perturbation method. The flexural-vibration of strip is unstable when the frequency of variable tension is two times as the natural frequency of flexural-vibration strip. The characteristic of current in a temp driving motor's main loop has been studied and tested, it has been proved that there are 6 harmonic component and 12 harmonic component in main loop of driving motor electricity. The vertical vibration of working roller has been tested, the test result approves that the running unsteady is caused by parametric vibration. It attaches importance to the parametric vibration of rolling mill.

[Key words] rolling mill; parametric vibration; Floquet theory
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1 INTRODUCTION

A vibration caused by a rolling mill not only affects the smoothness and steadiness of the running mill, but also affects the quality of the rolled strip, specially the surface quality. For a running rolling mill, sometimes it shows violent vibration and strip stripe, leading to closing down or breakdown of machine. In a field test, we find this kind of phenomenon by accidental, at a certain value of rolling mill tension, the vertical vibration of rolling mill is very strong. However, for some unknown reason, the tension changes greatly, as a result, the vertical vibration weakens rapidly, and the mill runs steadily. There are many reasons that cause violent vibration of the rolling mill; from energy viewpoint, the reason comes from two aspects mainly. The first one is that the rolling mill absorbs energy from the main drive system continuously, the vibration aggravates continuously, finally leads to unstability, however this aspect we can not explain the phenomenon that we observed. The second one is that through the strip the rolling mill gains energy from the coiling and uncoiling machine continuously, leads to the vibration strengthening continuously, the consequence is that the machine closing down or is destroyed. For unsteady of rolling mill caused by the first aspect reason, experts have undertaken a series of research, and have achieved a lot of valuable achievements[1~10]. For examples, the dynamics specific property analysis of the rolling system[1, 2], the 3 times frequency self-excited vibration analysis of the rolling system[3], study on characteristics of a bending vibration of rolling system and 5 times sound interval oscillation of a skin miller was conducted by Nester's group[4]. But for the unsteady caused by the second reason, few study has been done in the past, analysis of mechanism even less. It's necessary for us to do research on its mechanism and provide theoretical guidance for practical production.

In this paper, the authors establish a flexural-vibration model of a mill strip under the effect of the draught tension, analyze the influence about the rotational speed change of the coiling and uncoiling on the vibration of strip and the vibration of the mill system, thus explain the phenomenon that we examined from the aspects of both theory of test.

2 PARAMETER VIBRATION MODEL OF ROLLING MILL

The flexural-vibration of strip is caused by the tensions of the coiling and uncoiling and the vertical force. Establish the tension as F, the dynamics equation is

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} - F(t)\frac{\partial^2 w}{\partial x^2} + \frac{\rho h}{D}\cdot\frac{\partial^2 w}{\partial t^2} = f(t) \quad (1)$$

where w is the bending deflection of the strip; ρ density; h is thickness, $D = Eh^3/12(1-\mu^2)$ bending rigidity; E is the module of elasticity; μ is the Poisson ratio.

The free vibration equation is

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} - F(t)\frac{\partial^2 w}{\partial x^2} +$$

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$$\frac{ph}{D}\cdot\frac{\partial^2 w}{\partial t^2}=0 \tag{2}$$

The two ends (coiling and uncoiling end) of the strip (plate) can be looked as free-supported, while the other two-ends are free ends. The deflection of strip changes little in the direction of width. In order to investigate the vibration of the strip analytically, we suppose w is only a function of x and t, it means that the original solution is as

$$w=\overline{w(x,\ t)} \tag{3}$$

Let: $\overline{w(x,\ t)}=u(t)\sin(n\pi x/l)$, boundary condition is free-supported, we can induce:

$$\left[\frac{n^4\pi^4}{l^4}u(t)+F(t)\frac{n^2\pi^2}{l^2}u(t)+\frac{ph}{D}\ddot{u}(t)\right]\sin\frac{n\pi x}{l}=0 \tag{4}$$

Eqn.(4) can be established only when the parameter of $\sin(n\pi x/l)$ is equal to zero at any time. So we know that $u(t)$ fits differential equations:

$$\ddot{u}(t)+\tilde{p}_{0n}^2\left[1+\frac{F(t)}{p_{kn}}\right]u(t)=0$$
$$(n=1,\ 2\cdots) \tag{5}$$

where $p_{kn}=n^2\pi^2 D/l^2 ph$

Eqn.(5) is Hill equation. Then we have

$$\tilde{p}_{0n}=\frac{n^2\pi^2}{l^2}\sqrt{\frac{D}{ph}} \tag{6}$$

where $\tilde{p}_{0n}$ is the natural frequency of bending vibration of the strip with no tension.

During the process of the rolling, the tensions of coiling and uncoiling fluctuates thinly near a certain fixed value F_0, the wave frequency is revolving frequency of coiling and uncoiling, so we can get

$$F=F_0-\tilde{F}_0\cos\omega t \tag{7}$$

where ω is revolving frequency of coiling and uncoiling. Put Eqn.(7) into Eqn.(5), we can get

$$\ddot{u}(t)+\tilde{p}_{0n}^2\left[1+\frac{F_0-\tilde{F}\cos\omega t}{p_{kn}}\right]u(t)=0$$
$$(n=1,\ 2\cdots) \tag{8}$$

Suppose

$$p_{0n}=\tilde{p}_{0n}\sqrt{1+\frac{F_0}{p_{kn}}} \tag{9}$$

when $\tilde{F}\cos\omega t$ is a slight harmonic force, Eqn.(8) can be turned into Mathieu equation, that is

$$\ddot{u}(t)+p_{0n}^2[1-\tilde{h}\cos\omega t]u(t)=0 \tag{10}$$

where $\tilde{h}=\tilde{F}/p_{kn}$

3 PARAMETRIC VIBRATION STEADY CHARACTER OF ROLLING MILL

Suppose

$$\delta=p_{0n}^2\left(\frac{2}{\omega}\right)^2,\quad \varepsilon=p_{0n}^2\left(\frac{2}{\omega}\right)^2\tilde{h}$$

let $t=2t_1/\omega$, after put it into Eqn.(10), still marking symbol t_1 as t, we can obtain

$$\ddot{u}(t)+[\delta+\varepsilon\cos 2t]u(t)=0 \tag{11}$$

where $\varepsilon\ll 1$

According to Floquet theory[11], the canonical form of the Mathieu equation (Eqn.(11)) is

$$u(t)=\exp(\gamma t)\Phi(t) \tag{12}$$

where $\Phi(t)$ is a periodic function, its period is π or 2π, whether γ is real number or imaginary number is determined by the values of δ and ε. Based on Floquet theory, the boundaries between stability and unstability regions can be decided by the circle solution of Eqn.(11). These boundaries can be obtained by asymptotic expansion according to following forms:

$$\delta=n^2+\varepsilon\delta_1+\varepsilon^2\delta_2+\cdots \tag{13}$$

$$u=u_0+\varepsilon u_1+\varepsilon_2 u_2+\cdots \tag{14}$$

where n is an integer that included zero. Put Eqn.(13) and Eqn.(14) into Mathieu Eqn.(11), use the condition that the coefficient of the same power equation is zero, we can get

$$\ddot{u}_0+n^2u_0=0 \tag{15a}$$

$$\ddot{u}_1+n^2u_1=-(\delta_1+\cos 2t)u_0 \tag{15b}$$

$$\ddot{u}_2+n^2u_2=-(\delta_1+\cos 2t)u_1-\delta_2 u_0 \tag{15c}$$

Then we can get the solution of zero order equation (Eqn.(15a)):

$$u_0=\begin{cases}\cos nt\\ \sin nt\end{cases}\quad n=0,\ 1,\ 2\cdots \tag{16}$$

After we decide the solution of high order asymptotic solution under the condition of $n=0$, 1 and 2, it can be get from perturbation theory[12] that:

1) When $n=0$

$$\delta=-\frac{1}{8}\varepsilon^2+O(\varepsilon^2) \tag{17}$$

2) When $n=1$

From Eqn.(16) and suppose $x_0=\cos t$, we can get the boundary of the regions of stability and unstability in the δ-ε trace:

$$\delta=1-\frac{1}{2}\varepsilon-\frac{1}{32}\varepsilon^2+O(\varepsilon^3) \tag{18}$$

If we suppose $x_0=\sin t$, the another boundary is

$$\delta=1+\frac{1}{2}\varepsilon-\frac{1}{32}\varepsilon^2+O(\varepsilon^3) \tag{19}$$

3) When $n=2$

Also from Eqn.(16) and establish $x_0=\cos t$, we can get the boundary of the regions of stability and unstability in the δ-ε trace:

$$\delta=4+\frac{5}{48}\varepsilon^2+O(\varepsilon^3) \tag{20}$$

If we suppose $x_0=\sin t$, the other boundary is

$$\delta=4-\frac{5}{48}\varepsilon^2+O(\varepsilon^3) \tag{21}$$

The boundaries of Eqns.(17) ~ (21) of the regions of stability and unstability in the δ-ε trace are shown in Fig. 1, in which the shadow region in the boundary is unstability, the other region is stability.

4 UNSTABLE CONDITION OF VIBRATION OF STRIP

The unstable condition of vibration of strip is induced from Eqns. (18) and (19), when

$$|\delta - 1| = \frac{1}{32}\varepsilon^2 \tag{22}$$

the vibration of strip is unstable.

Put $\delta = p_{0n}^2(2/\omega)^2$ into Eqn. (22), we can get

$$\delta = p_{0n}^2(2/\omega)^2 \cong 1 \tag{23}$$

$$p_{0n} \cong \omega/2 \tag{24}$$

Eqn. (24) indicates that the flexural-vibration of strip is unstable if the frequency of variable tension is two times as the natural frequency of strip flexural-vibration. It is induced that when the vertical-vibration of mill aggravates, the mill system is unstable.

With the same reason, it can be induced from Eqns. (20) and (21), when

$$|\delta - 4| = \frac{1}{48}\varepsilon^2 \tag{25}$$

the vibration of strip is unstable.

Put $\delta = p_{0n}^2(2/\omega)^2$ into Eqn. (25), we can get

$$\delta = p_{0n}^2(2/\omega)^2 \cong 4 \tag{26}$$

namely

$$p_{0n} \cong \omega \tag{27}$$

Eqn. (27) indicates that the flexural-vibration of strip is unstable too if the frequency of variable tension is the same as the natural frequency of flexural-vibration strip, which leads to aggravate the mill vertical-vibration and unstable mill system.

5 ANALYSES OF LIVING SURVEY

In a certain large iron and steel enterprise, sometimes violent temp mill vibration occurs and stripes appear on the surface of strip, which seriously affects the normal production and the quality of products seriously. Entrusted by the enterprise, we have done a lot of field tests on the vibration of skin miller and the parameters of electrical system.

5.1 Electrical spectral analyses of coiling and uncoiling

The running equipments of the temp mill's coiling and uncoiling machine adopt the direct current electrical driving system which is consisted of SCR-D. The driving system uncoiling is a coaxial driving double-motors. Two running machines of the coiling machine are driven coaxially by steel-band couple. The rectiformer for the motors is a three-winding transformer, whose mode of connection is D/y5 and d0. Thus the phase difference between the input voltage of these two sets of rectifiers is 30°. In the waveform of commutating voltage, the 6th harmonic component is inverse (the phase difference is 180°), while the phase of the 12th harmonic component is the same. When magnetic flux is a constant, the 6th harmonic torque of the two electromachines is just right opposite in direction, and the 12th harmonic torque is just right the same in direction. The very character of the torque of running electromachine will influence the dynamics deeds of machine system; such influence is decided by the relationship of the machine system's parameter and the electrical parameter.

As the input voltage of the two sets of rectifier is in phase, great harmonic component in the main circuit of drive motor is caused. In fact the field tests show that the main circuit exists large harmonic component, shown in Fig. 1, the electrical circuit has following characters:

1) The feedback of the coiling and uncoiling motors and the signal of the disturbing noise are very weak, which has little influence to the torque and the force of coiling and uncoiling.

2) The harmonic component of the main circuit in the drive motor's circuit is very obvious, Figs. 1 and 2 shows the spectrum of the drive motor main circuit, the main harmonic component is the 6th harmonic (300 Hz), the 12th harmonic (600 Hz) and the 18th harmonic (900 Hz).

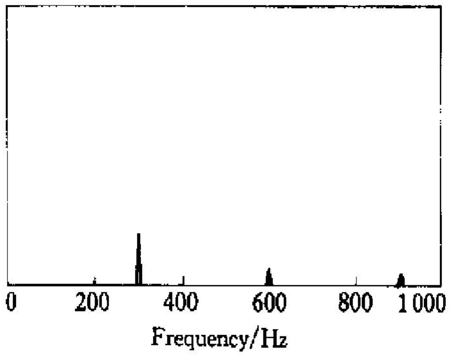

Fig. 1 Spectrum of coiling motor main current

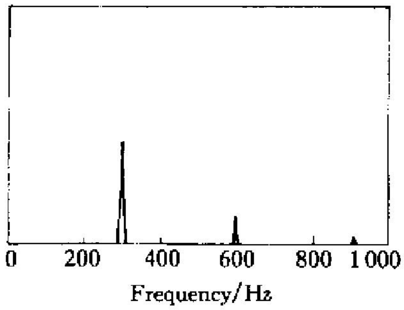

Fig. 2 Spectrum of uncoiling motor main current

3) The harmonic component of the drive motor excitation circuit is very small, the reason is that the inductance of excitation circuit is large, the character of the flat component rejects the harmonic current validly.

The above analysis and the field tests indicates that during the process of production, large harmonic component is caused in the drive motor main circuit. The main harmonic component are the 6th humorous (300 Hz), the 12th humorous (600 Hz) and the 18th humorous (900 Hz). The harmonic current causes to the change of the torque, with the same frequency as that of harmonic current, and the change of drive motor's torque results in the changes of coiling and uncoiling tensions, since the tension change is in accordance with the torque, the change frequency of tension is in accordance with the frequency of the harmonic current, i. e., the superposition of the circle function of 300, 600, 900 Hz. They can be expressed as

$$F = F_0 + \alpha_1 \cos(600\pi t) + \alpha_2 \cos(1200\pi t) + \alpha_3 \cos(1800\pi t) \quad (28)$$

where F is the tension of coiling or uncoiling, F_0 is the constant item of the changed coiling and uncoiling tension; α_i is constant.

5.2 Natural frequency of strip vibration and test of mill vertical-vibration

During the process of rolling, the rolling draught tension is determined by production process. Theoretically the tension is a constant, however, it's real fluctuated slightly. When the tension is F_0, from Eqn. (9) it can be induced that the natural frequency of strip flexural-vibration is

$$p_{0n} = \tilde{p}_{0n} \sqrt{1 + \frac{T_0}{p_{kn}}} \quad (29)$$

For example, when the skin miller draughts a certain steel-plate with the thickness of 1.98 mm, the tension is 73 kN, it can be calculated from Eqn. (29) that the third order natural frequency of strip flexural-vibration is 301.11Hz. This frequency value is close to half of the 6th harmonic current frequency of the motor main circuit current, or the frequency of the tension's 600 Hz component. From the above analysis of the unstable condition of strip flexural-vibration, we can get that the strip appears flexural-vibration unstable which strengthens the vertical-vibration of running mill, as a result the vertical-vibration of the rolling mill becomes violent.

The field test analysis proves the analysis above. Fig. 3 shows the spectrum of chart of running mill vertical-vibration. The set-off crest frequency of running mill vertical-vibration is closed to 600 Hz, as shown in Fig. 3, it is induced that the strip appears unstable flexural-vibration. Because the strip tension exists harmonic component of 600 Hz, and the third-order natural frequency (301.11 Hz) of the strip flexural-vibration is closed to half the harmonic component frequency of the strip tension (600 Hz), the strip flexural-vibration is unstable, which makes the upper and lower drive mill all vibrate violently in vertical direction.

From the above strip unstable condition, we can induce that when the natural frequency of strip flexural-vibration is close to half of the harmonic

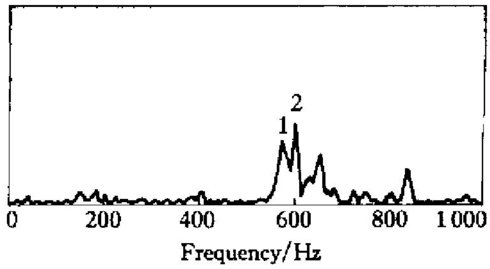

Fig. 3 Spectrum chart of running mill vertical-vibration

component frequency of the tension which changes periodically, the strip flexural-vibration becomes unstable; and the natural frequency of flexural-vibration relates to tension, therefore we can avoid unstability by the way of changing tension.

[REFERENCES]

[1] Nessier G L, Cory J Fr. Cause and solution of fifth octave backup roll chatter on 4-h cold mills and temper mills [J]. ALSE Year Book, 1989(12): 33-37.

[2] Gasparic J. Vibration analysis identifies the cause of mill chatter [J]. ALSE Year Book, 1991(2): 27-29.

[3] Bollinger L A. Winding reel involvement in temper mill chatter [J]. ALSE Year Book, 1994(11): 27-29.

[4] ZHONG Jue. The industrial test and discover of the band steel surface chatter mark [J]. The Chinese Journal of Nonferrous Metals, (in Chinese), 2000(10): 291-296.

[5] Wang Z. Dynamics characteristics of a rolling mill drive system with backlash in rolling slippage [J]. Journal of Materials Processing Technology, 1997(2): 128-133.

[6] Liou C H, Lin H H, Oswald F B, et al. Effect of contact ratio on spur gear dynamic load with No tooth profile modifications [J]. ASME Journal of Mechanical Design, 1996, 118: 439.

[7] Yun I S, Wilson W R D, Ehmann K F. Chatter in Rolling [M]. Trans of the NAMRI XXIII of SME, 1995, 108(1): 13-19.

[8] Brown R E, Maliotis G N. PID Self-Tuning controller for aluminum rolling mill [J]. IEEE Trans Industry Applications, 1993, 29(3): 578-583.

[9] TANG Hua-ping, YAN Hang-zhi, ZHONG Jue. Torsional self-excited vibration of rolling mill [J]. Trans Nonferrous Met Soc China, 2002, 12(2): 291-293.

[10] TANG Hua-ping, ZHONG Jue. Qualitative analysis of self-excited horizon vibration in single-roll driving mill system [J]. Chinese Journal of Mechanical Engineering, 2001, 37(8): 55-59.

[11] Guo R M. Stress analysis and life expectancy of rolling mill bouting [J]. Iron and Steel Engineer, 1992, 69(2): 45-53.

[12] ZHENG Zhao-chang. The Machinery Vibration (The Middle Volume) [M]. Beijing: The Mechanical Industry Press, 1986.

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