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Conformal mapping modeling of metal plastic deformation and die cavity in special-shaped extrusion①

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[Abstract] With the help of Complex Function Mapping studied results, the analysis function of Conformal Mapping is set up. Since the complicated three dimension's deformation problems are transferred into two dimension problems, both the stream function and strain ratio field are analyzed in the metal plastic deformation. Using the upper-bound principles, the theory of metal deformation and die cavity optimized modeling is established for random special-shaped product extrusion. As a result, this enables the realization of intelligent technique target in the die cavity of CAD/CAM integration.

[Key words] special-shaped products; extrusion; die cavity; conformal mapping

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1 INTRODUCTION

In order to meet the technologic request in quickly precise processing for extrusion of special-shaped metal products, how to realize the intelligent technique in metal special-shaped extruding has been an important task[1]. For non-circle section extrusion, because of the existence of non-neglecting tangent direction fluent of metal plastic, the deformation theory analysis becomes more complicated[2]. Lacking of exact mathematics theory support, the three dimension shaping problems turn to difficulty in solution, and the research makes slow progress, which leads to no universally systematical theory in special-shaped extrusion[3, 4]. Even if some results were reported[5], the results are all focused on specific special-shaped product but can not be used generally and systematically, which causes the difficulty of application in random extruding modeling as well as the realization of CAD/ CAM technique target in highly accurate surface of die cavity. In this paper, with the help of Complex Function Mapping studied results[6, 7], the authors aim at setting up the analysis function of Conformal Mapping which can turn arbitrary region into unit dish so that the complicated three dimension plastic flow problems can transform into two dimension problems and can analyze both the stream function and strain ratio field in the metal plastic deformation. Also, the authors want to establish both the optimized mathematic model of die cavity and three dimension's theory of metal deformation for random special-shaped product extrusion by using the upper-bound principles[8, 9]. This can provide a theoretical basis to achieve the intelligent technique target of CAD/CAM integration for exact metal extruding and the die cavity manufacture; meanwhile, make the realization in engineering and technology possible.

2 MODELING OF SPECIAL-SHAPED SECTION AND DIE CAVITY

The key factor of metal plastic deformation modeling lies on whether the geometrical boundary and region of special-shaped section can be described as function systematically. Based on Complex Function Mapping theory, the complex polynomial function can describe the complicated geometrical region of special-shaped product as follows:

$$W = \sum_{n=1}^{\infty} c_n \xi^n \quad (\xi = \rho\exp(\mathrm{i}\theta)) \tag{1}$$

where ρ and θ are modulus and phase of the complex vector ξ in unit dish region respectively, and $c_n = a_n + \mathrm{i}b_n$ is the complex coefficient of polynomial.

During analyzing c_n in Eqn. (1), to simplify the calculation of modeling, the mapping function is expressed by triangle function tracking the Complex characters. Then the following function of Conformal Mapping region is gotten:

$$\begin{cases} x = \sum_{n=1}^{\infty} c_n \rho^n \cos n\theta \\ y = \sum_{n=1}^{\infty} c_n \rho^n \sin n\theta \end{cases} \tag{2}$$

In order to guarantee the high accuracy c_n, the interpolated normal convergence method has been set up by the authors. Above analysis method is named as Triangle Interpolation Method. As a result, the complicated region (as shown in Fig. 1(b)) can be

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pictured by unit dish region(as shown in Fig. 1(a)).

The a_n and b_n can be gotten:

$$\begin{cases} a_n = \dfrac{1}{N}\sum_{k=1}^{N}[x_k\cos n\theta_k + y_k\sin n\theta_k] \\ b_n = \dfrac{1}{N}\sum_{k=1}^{N}[-x_k\sin n\theta_k + y_k\cos n\theta_k] \end{cases} \quad (3)$$

$(n = 1, 2, \cdots, N)$

When geometrical boundary conditions in sections of inlet billet and outlet products are fulfilled, as shown in Fig. 2, the curved surface function of three-dimension die cavity is

$$R(\theta, z) = f(z)R_i + [1 - f(z)]R(\theta, L) \quad (4)$$

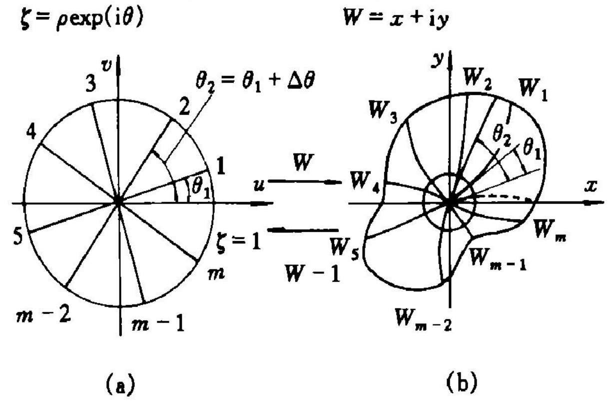

Fig. 1 Conformal mapping between complicate region and unit dish

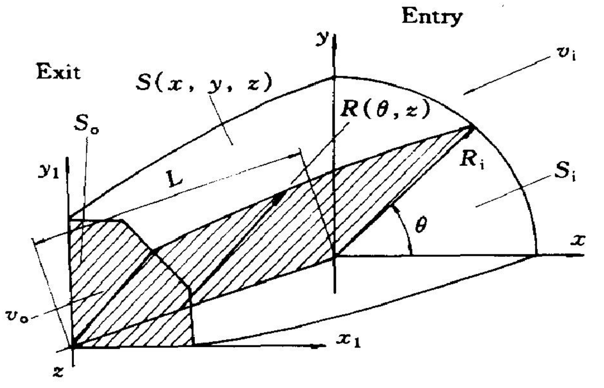

Fig. 2 Special-shaped extrusion

From Eqn. (2) and Eqn. (4), in the deformation zone, parameter function of metal plastic region is

$$\begin{cases} x = R_i f(z)\rho\cos\theta + [1 - f(z)]\sum_{n=1}^{N} c_n\rho^n\cos n\theta \\ y = R_i f(z)\rho\sin\theta + [1 - f(z)]\sum_{n=1}^{N} c_n\rho^n\sin\theta \\ z = z \end{cases}$$

$$(0 \le z \le L, \ 0 \le \theta \le 2\pi, \ 0 \le \rho \le 1) \quad (5)$$

From above statement, over the methods of Complex Mapping, triangle interpolation, polynomial interpolation and the normal convergence, the die cavity of exact extrusion has been modeled(when $\rho = 1$).

3 METAL PLASTIC FLOW FIELD FOR SPECIAL-SHAPED EXTRUSION

As shown in Fig. 2, for metal die, S_i is the area of entry cross-section; S_z is the area of random cross-section in deformation region; v_i presents plastic flow velocity of metal billet along z-axis direction on entry cross-section. Velocity v_z in the axial direction on any cross-section $z = z$ is supposed constant and is defined by

$$v_z = \frac{S_i}{S_z}v_i \quad (6)$$

In the process of axisymmetric extruding deformation, from the inverse Conformal Mapping function of Eqn. (5), we can get two stream surface functions in metal plastic flow:

$\rho(x, y, z) =$ Constant; $\theta(x, y, z) =$ Constant (7)

Since the relationship between axisymmetric extruding deformation region and special-shaped region is one-to-one mapping, then one-to-one correspondence relationship can also be gotten between the flow surface function in special-shaped product plastic field and the flow surface function Eqn. (7) of axis-symmetry. Eqn. (7) is turned into total differential equation based on ρ and θ respectively(ρ and θ represent two of the any flow surface functions) :

$$\begin{cases} \dfrac{\partial\rho}{\partial x}dx + \dfrac{\partial\rho}{\partial y}dy + \left[\dfrac{\partial\rho}{\partial x}\dfrac{\partial x}{\partial z} + \dfrac{\partial\rho}{\partial y}\dfrac{\partial y}{\partial z}\right]dz = 0 \\ \dfrac{\partial\theta}{\partial x}dx + \dfrac{\partial\theta}{\partial y}dy + \left[\dfrac{\partial\theta}{\partial x}\dfrac{\partial x}{\partial z} + \dfrac{\partial\theta}{\partial y}\dfrac{\partial y}{\partial z}\right]dz = 0 \end{cases} \quad (8)$$

From Eqn. (8), the streamline function of special-shaped product plastic flow is gotten:

$$\frac{dx}{-\sum_{n=1}^{\infty}A'_n(z)\rho^n\cos n\theta} = \frac{dy}{-\sum_{n=1}^{\infty}A'_n(z)\rho^n\sin n\theta} = \frac{dz}{1} \quad (9)$$

where $A_1(z) = R_i f(z) + a_1[1 - f(z)]$,
$A_n(z) = a_n[1 - f(z)] \quad (n = 2, 3, \cdots, N)$

Deducing the following result from hydrodynamics theory:

$$\frac{dx}{v_x} = \frac{dy}{v_y} = \frac{dz}{v_z} \quad (10)$$

Within the deformation zone of metal special-shaped extrusion, assume that v_x, v_y and v_z are the velocity components along x, y and z axial directions respectively. Put Eqn. (9) into Eqn. (10), the stream function of special-shaped extrusion is gotten:

$$\frac{v_x}{\sum_{n=1}^{\infty}A'_n(z)\rho^n\cos n\theta} = \frac{v_y}{-\sum_{n=1}^{\infty}A'_n(z)\rho^n\sin n\theta} = \frac{v_z}{1} \quad (11)$$

The strain rate field is inferred by plastic deformation principles[10]:

$$\dot{\varepsilon}_{ij} = \frac{1}{2}\left[\frac{\partial v_j}{\partial x_i} + \frac{\partial v_i}{\partial x_j}\right] \quad (12)$$

As shown in Fig. 2, suppose that $S(x, y, z)$ is the function of die cavity surface, Eqn. (11) and Eqn. (12) namely velocity field and strain rate field can be proved as it satisfies the velocity boundary conditions and the incompressible conditions of volume:

$$\mathbf{grad}S(x, y, z)\cdot \boldsymbol{v}_{\rho=1}=0;\ \dot{\varepsilon}_x+\dot{\varepsilon}_y+\dot{\varepsilon}_z=0 \tag{13}$$

From conformal mapping functions Eqn. (5) and Eqn. (7), the mutual transformation of metal plastic stream surface functions between axisymmetric extruding and special-shaped products can be gotten, then the mutual transformation of plastic flow field can be analyzed between three dimension special-shaped extrusion and axisymmetric extrusion.

4 ENERGY DISSIPATION RATES AND OPTIMIZED PARAMETER OF DIE

Suppose that σ_s means the yield stress or the flow stress of metal material, $\dot{\bar{\varepsilon}}$ means the effective strain rate, V presents the volume of metal deformation region, k means the yield shear stress, m means friction factor, $v_{S_0(z=0)}$, $v_{S_i(z=L)}$ is discontinue velocity paralleling to two cross-sections at entry and at the exit respectively, and $v_{S(\rho=1)}$ presents relative velocity between die cavity and metal surface, then the total energy dissipation rate is equal to the sum of internal energy dissipation rate in deformation zone, the energy dissipation rates on velocity discontinuity surfaces S_i and S_0 and the friction energy dissipation rates on the die-metal surfaces S, respectively.

From the upper-bound principle, in metal plastic deformation zone, the total energy dissipation rate

$$J^* = \iiint \dot{\bar{\varepsilon}}\sigma_s \mathrm{d}V + k\iint_{S_i} v_{S_i(z=0)}\mathrm{d}S_i + k\iint_{S_0} v_{S_0(z=L)}\mathrm{d}S_0 + mk\iint_{S} v_{S(\rho=1)}\mathrm{d}S \tag{14}$$

J^* is Suppose P is the relative pressure ratio,

$$P=\frac{J^*}{S_i v_{i(z=0)}\sigma_s} \tag{15}$$

Asking P for extremum in Eqn. (15), the optimized parameter L, an admissible velocity field and strain rate field can be gotten then CAD gets realized. Combining this with current NC techniques, such as UG and Pro/E software Numerical Control module, which enable the realization of intelligent technique target in the CAD/CAM of die cavity.

5 EXAMPLE

As shown in Fig. 3, the product belongs to hexagon, a presents the length of each side, r is the arc radius, and geometrical center is (0, 0). Here we choose the first quadrant to analyze the hexagon. From the dependence of geometrical boundary conditions, the boundary function is gotten:

$$\begin{cases} y=\frac{\sqrt{3}}{3}a \\ 0< x< \frac{a}{2}-\frac{\sqrt{3}}{3}r \\ \left[x-\left(\frac{a}{2}-\frac{\sqrt{3}}{3}r\right)\right]^2+\left[y-\left(\frac{\sqrt{3}}{2}a-r\right)\right]^2=r^2 \\ \frac{a}{2}-\frac{\sqrt{3}}{3}r< x< \frac{a}{2}+\frac{\sqrt{3}}{6}r \\ y=-\frac{\sqrt{3}}{2}(x-a)r \\ \frac{a}{2}+\frac{\sqrt{3}}{6}r< x< a-\frac{\sqrt{3}}{6}r \\ \left[x-\left(a-\frac{2\sqrt{3}}{3}r\right)\right]^2+y^2=r^2 \\ \frac{a}{2}-\frac{\sqrt{3}}{6}r< x< a-\left(\frac{2\sqrt{3}}{3}-1\right)r \end{cases} \tag{16}$$

Suppose that $a=1$ and $r/a=0.15$, over the numerical and iterative calculation of triangle interpolation ($N=32$ interpolating points, which represent the infinite points of Eqn. (2) in the first quadrant), according to Ref. [11], c_n can be calculated:

$a_{2j-1}=$ 0.897 094, 0.000 073, 0.000 343, 0.039 462, 0.000 421, 0.000 030, 0.011 113, 0.000 064, 0.000 383, 0.004 214, − 0.000 620, 0.000 180, 0.002 425, − 0.000 049, − 0.002 354, 0.000 270

$a_{2j}=0$; $b_{2j}=b_{2j-1}=0$ $(j=1, 2, \cdots, 16)$

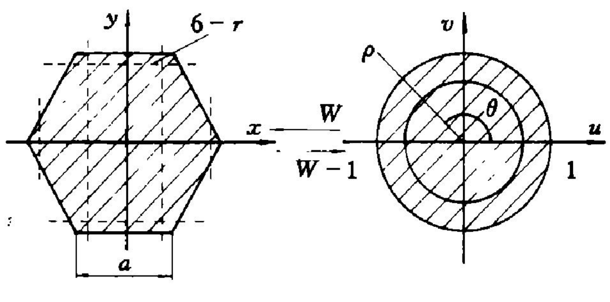

Fig. 3 Conformal mapping between hexagon region and unit dish

Assume that the material is not work hardening, put the diameter $d=2.3a$ of the circular rod into Eqn. (15), then by optimizing the length L in metal deformation region, functions of die surface and deformation region are gotten. For example, as shown in Fig. 4, minimum P point on curve 3 relates to optimal parameter of hexagon die, $L/a=1.20$. When the friction conditions keep constantly on the interface between metal and die, with the enlarged reduction ratio λ in area, P extremum will increase, and parameter L/a has an increasing trend. At the same time, Eqn. (4) will realize the CAD of hexagon die (as shown in Fig. 5(a)).

Fig. 5(b) is an optimal die figure of leaf-shaped product (the geometrical parameter of the product

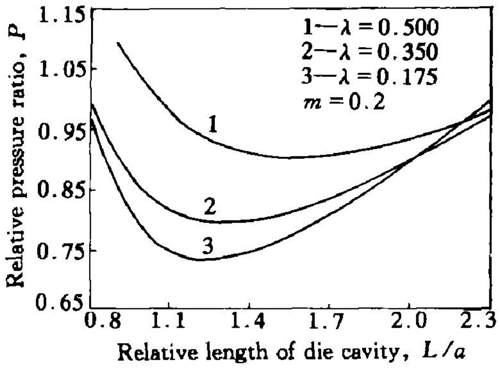

Fig. 4 Optimized parameter curve of die cavity

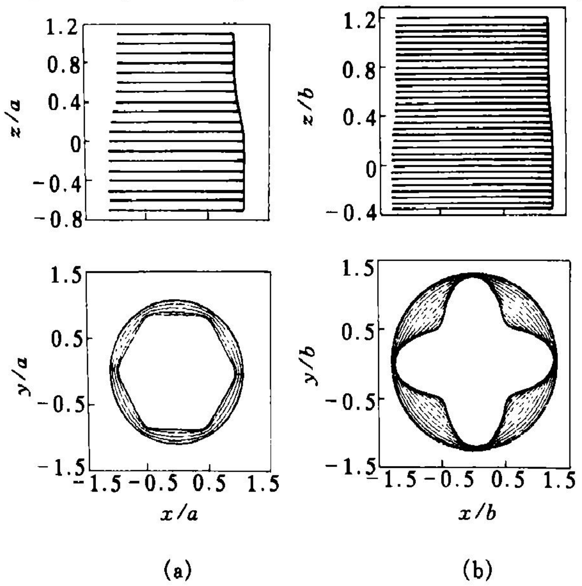

Fig. 5 Special-shaped dies
(a) —Hexagon-shaped die;
(b) —Leaf-shaped die

cross-section, $b = 10$ and $r = 1.15b$) when $\lambda = 0.35$, $m = 0.2$ and the diameter of the circular rod is $2.5\ b$.

Above theoretical analysis method can be applied to other kinds of special-shaped products in optimal parameters of die.

6 CONCLUSIONS

1) From Conformal Mapping theories for metal deformation and the optimized die, both mathematics models have been gotten: one is flow field of metal deformation, the other is three dimensions surface of die cavity. Meanwhile, for special-shaped products, the prediction of optimal die parameter has realized under different deformation conditions.

2) By combining the established die cavity model with Numerical Control software, the precise manufacture CAM for three dimension surface can be achieved.

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