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Non-derivative solution to nonlinear dynamic optimal design of class two for deformation network monitoring①

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[Abstract] Based on the nonlinear error equation of deformation network monitoring, the mathematical model of nonlinear dynamic optimal design of class two was put forward for the deformation network monitoring, in which the target function is the accuracy criterion and the constraint conditions are the network's sensitivity, reliability and observing cost. Meanwhile a new non-derivative solution to the nonlinear dynamic optimal design of class two was also put forward. The solving model uses the difference to stand for the first derivative of functions and solves the revised feasible direction to get the optimal solution to unknown parameters. It can not only make the solution to converge on the minimum point of the constraint problem, but decrease the calculating load.

[Key words] deformation network monitoring; nonlinear dynamic optimal design; non-derivative analytic method.
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1 INTRODUCTION

The optimal design for deformation network monitoring is one of head fields studied at abroad and home. Four study objects were put forward by FIG in 1990, one of them is the optimal design for modern deformation network monitoring. The quality and cost of deformation network monitoring are mainly dependent on the design. Therefore the design theory of deformation network monitoring is one of main contents of modern deformation monitoring theories. The optimal design of class two first put forward by Grafarend is the design for observing plan and weight to control observing accuracy and cost of deformation network monitoring, which is the importance of optimal designs of four classes. The optimal design of deformation network monitoring as a scientific and strict method, is different from the classical standard design, because the most functions of deformation monitoring system are nonlinear. Obviously the nonlinear dynamic optimal design for deformation network monitoring is more scientific and accurate than the linear programming design widely used.

TAO et al earlier put forward the nonlinear dynamic optimal model of class two and its solution method for deformation network monitoring, which makes the target function $f(\boldsymbol{P})$ to be minimum under the constraint condition $g(\boldsymbol{P}) \geqslant 0$ to get the optimal observing weight[1]. All solution methods must analytically calculate the first derivative of the target function[2,3]. Then the feasible descent direction $\boldsymbol{S}^{(K)}$ at the feasible point $\boldsymbol{P}^{(K)}$ must be determined using the derivative of the target function. The pace factor α_K can be calculated in the direction $\boldsymbol{S}^{(K)}$ with the linear seeking method. Then we can get a new point $\boldsymbol{P}^{(K+1)}$. The key problem of the method is how to calculate the seeking direction. Calculating the first derivative is necessary to repeatedly solve the seeking direction $\boldsymbol{S}^{(K)}$ every time, whose working load is very large. Based on these, the method put forward in the paper uses the difference to stand for the first derivative of function and repeatedly solves the optimal result in the revised feasible direction. The analytic method has the same convergence as the existing methods, for it makes the result to better converge on the minimum of the constraint problem[4,5]. In the meantime, the analytic method can largely decrease the working load. It is simple and practical to calculate the result with the method.

2 QUALITY CRITERIA AND MATHEMATICAL MODEL

TAO et al have given the nonlinear error equation of deformation network monitoring as follows[1]:

$$\left.\begin{array}{ll} V_1 = \phi_1(d_1, d_2, \cdots, d_m) - l_1 & P_1 \\ V_2 = \phi_2(d_1, d_2, \cdots, d_m) - l_2 & P_2 \\ \vdots \quad\quad \vdots & \vdots \\ V_n = \phi_n(d_1, d_2, \cdots, d_m) - l_n & P_n \end{array}\right\} \quad (1)$$

where $\boldsymbol{P}$ is a diagonal weight matrix of measurement, $\boldsymbol{P}^{\mathrm{T}} = (P_1, P_2, \cdots, P_n)$; l_i is measurement whose correction is V_i, d_i is the displacement of deformation parameter. Therefore we can obtain all quality criteria as follows:

$$\boldsymbol{V} = \phi(\boldsymbol{d}) - \boldsymbol{l} \qquad \boldsymbol{P} \quad (2)$$

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The covariance K_{dd} of displacement d of deformation parameter is known as the accuracy criterion, that is[6]

$$K_{dd} = \left[A^{T} PA - \bar{Y}^{T} \frac{\partial f_s'}{\partial d} \right]^{-1} A^{T} PA \cdot \left[\left[A^{T} PA - \bar{Y}^{T} \frac{\partial f_s'}{\partial d} \right]^{-1} \right]^{T} \tag{3}$$

where

$$f_s' = \left[\frac{\partial \phi_1}{\partial d_1}, \frac{\partial \phi_2}{\partial d_1}, \cdots, \frac{\partial \phi_n}{\partial d_1}, \frac{\partial \phi_1}{\partial d_2}, \cdots, \frac{\partial \phi_n}{\partial d_2}, \cdots, \frac{\partial \phi_n}{\partial d_m} \right]^{T},$$

$$\bar{Y}^{T} = \begin{bmatrix} y_1 & y_2 & \cdots & y_n & 0 & 0 & & & & \cdots & & & 0 \\ & & & 0 & y_1 & y_2 & \cdots & y_n & & \cdots & & & 0 \\ \vdots & & & & & & & & & & & & \vdots \\ 0 & & & & \cdots & & & & 0 & y_1 & y_2 & \cdots & y_n \end{bmatrix},$$

$$A = \frac{\partial \phi}{\partial d}, \qquad y_i = -P_i(\phi_i(d) - l_i).$$

The critical value to find out the displacement can be called as the measure of sensitivity of deformation network monitoring, that is

$$\nabla d_0 = \frac{\sigma_0 \delta_0}{\sqrt{g^{T} Q_d^{-1} g}} \tag{4}$$

where σ_0 is the variance of unit weight; δ_0 is the non-central parameter, in general $\delta_0 = 4.3$[7]; g can be determined by the pre-given direction. To simplify the calculation, let $Q_d = 2(A^{T} PA)^{-1}$. Then Eqn. (4) can be rewritten as follows:

$$\nabla d_0 = \frac{\sigma_0 \delta_0}{\sqrt{\frac{1}{2} g^{T} A^{T} PAg}} \tag{5}$$

The reliability criterion of network is the minimum of gross errors which can be found out, that is[8]

$$\nabla_0 l_i = \frac{\sigma_0 \delta_0}{P_i \sqrt{q_{ii}}} \tag{6}$$

The observing cost criterion is that the sum of observing weights is equal to or less than a given value ω, that is [9]

$$\delta \bar{P} \leqslant \omega \tag{7}$$

where $\delta = (1, 1, \cdots, 1)$, $\bar{P} = (P_1, P_2, \cdots, P_n)^{T}$.

The unknown parameter is the observing weight P in the mathematical model of nonlinear dynamic optimal design of class two for deformation network monitoring. The target function $f(P)$ is the covariance K_{dd} of the displacement of the deformation parameter, that is

$$f(P) = K_{dd} = \left[A^{T} PA - \bar{Y}^{T} \frac{\partial f_s'}{\partial d} \right]^{-1} \cdot A^{T} PA \left[\left[A^{T} PA - \bar{Y}^{T} \frac{\partial f_s'}{\partial d} \right]^{-1} \right]^{T} = \min \tag{8}$$

The constraint condition is given as following:

The sensitivity constraint is

$$\nabla d_0 - \frac{\sigma_0 \delta_0}{\sqrt{\frac{1}{2} g^{T} A^{T} PAg}} \geqslant 0$$

where ∇d_0 is the minimum displacement pre-given in design stage.

The reliability constraint condition is

$$\nabla_0 l_i - \frac{\sigma_0 \delta_0}{P_i \sqrt{q_{ii}}} \geqslant 0 \tag{9}$$

where $\nabla_0 l_i$ is the pre-given threshold value to probe the gross error in design stage.

The cost constraint is

$$\begin{bmatrix} -1 & -1 & & \cdots & -1 \\ 1 & 0 & & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & & & & \vdots \\ 0 & 0 & & \cdots\ 0 & 1 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_n \end{bmatrix} - \begin{bmatrix} -\omega \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \geqslant 0 \tag{10}$$

which can be expressed as follows:

$$E\bar{P} - b \geqslant 0$$

where

$$E = \begin{bmatrix} -1 & -1 & & \cdots & -1 \\ 1 & 0 & & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & & & & \vdots \\ 0 & 0 & & \cdots\ 0 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} -\omega \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}.$$

Therefore we can get the constraint set of the problem as following

$$g_1(P) = \nabla d_0 - \frac{\sigma_0 \delta_0}{\sqrt{\frac{1}{2} g^{T} A^{T} PAg}} \geqslant 0,$$

$$g_2(P) = \nabla_0 l_i - \frac{\sigma_0 \delta_0}{P_i \sqrt{q_{ii}}} \geqslant 0,$$

$$g_3(P) = E\bar{P} - b \geqslant 0.$$

Then we can get the optimal design model (M)

$$(\mathrm{M}) \qquad \begin{matrix} \text{obj.} & \min f(P) \\ \text{s.t.} & g_1(P) \geqslant 0 \\ & g_2(P) \geqslant 0 \\ & g_3(P) \geqslant 0 \end{matrix} \tag{11}$$

Suppose P is the optimal solution to the optimal problem (M) and $P^{(K)}$ is the feasible solution that is very close to P. If $P^{(K)}$ can converge on the optimal solution of the constraint problem, $P^{(K)} - P = \alpha_K S^{(K)}$. But how to calculate the seeking direction

$S^{(K)}$ is the key problem. We can get[10]

$$B^{(K)} S^{(K)} = -\nabla f(P^{(K)}) \quad (12)$$

where $B^{(K)}$ is a positive definite and symmetrical matrix which only contains the information of the first derivative; $\nabla f(P^{(K)})$ is the first derivative matrix. To solve Eqn.(12), we can obtain the direction $S^{(K)}$, and analytically calculate the first derivative Eqn.(12). Although Eqn.(12) does not contain the second derivative, we must calculate the first derivative when repeatedly solving Eqn.(12). Therefore this paper puts forward the difference to stand for the first derivative to simplify the calculation.

Let $f(P) = f(P_1, P_2, \cdots, P_n)$, if $\| P^{(K)} - P \| \leqslant \varepsilon$, we can get

$$\lim_{\substack{\varepsilon \to 0 \\ P^{(K)} \to P}} \left[\frac{f(P_1^{(K)} + \varepsilon, P_2^{(K)}, \cdots, P_n^{(K)},)}{\varepsilon} - \frac{f(P_1^{(K)}, P_2^{(K)}, \cdots, P_n^{(K)})}{\varepsilon} \right] = \nabla_{P_1} f(P)$$

whose first difference is

$$\Delta f_1 = f(P_1^{(K)} + \varepsilon, P_2^{(K)}, \cdots, P_n^{(K)}) - f(P_1^{(K)}, P_2^{(K)}, \cdots, P_n^{(K)}).$$

If $\varepsilon \to 0$,

then $\Delta f_1 \to \nabla_{p_1} f(P)$.

Hence we can get

$$\Delta f_1 \approx \nabla p_1 f(P).$$

The same as the above, we can obtain

$$\Delta f_2 = f(P_1^{(K)}, P_2^{(K)} + \varepsilon, P_3^{(K)}, \cdots, P_n^{(K)}) - f(P_1^{(K)}, P_2^{(K)}, \cdots, P_n^{(K)}) \approx \nabla_{p_2} f(P),$$

$$\vdots$$

$$\Delta f_n \approx \nabla_{p_n} f(P).$$

We can also get the differences of $g_1(P)$, $g_2(P)$ and $g_3(P)$ as following:

$$\Delta g_{11} = g_1(P_1^{(K)} + \varepsilon, P_2^{(K)}, \cdots, P_n^{(K)}) - g_1(P_1^{(K)}, \cdots, P_n^{(K)}) \approx \nabla_{p_1} g_1(P^{(K)}),$$

$$\Delta g_{12} \approx \nabla_{p_2} g_1(P^{(K)}),$$

$$\vdots$$

$$\Delta g_{1n} = \nabla_{p_n} g_1(P^{(K)});$$

$$\Delta g_{21} \approx \nabla_{p_1} g_2(P^{(K)}),$$

$$\Delta g_{22} \approx \nabla_{p_2} g_2(P^{(K)}),$$

$$\vdots$$

$$\Delta g_{2n} = \nabla_{p_n} g_2(P^{(K)});$$

$$\Delta g_{31} \approx \nabla_{p_1} g_3(P^{(K)}),$$

$$\Delta g_{32} \approx \nabla_{p_2} g_3(P^{(K)}),$$

$$\vdots$$

$$\Delta g_{3n} = \nabla_{p_n} g_3(P^{(K)}).$$

Finally we can get

$$\Delta f = (\Delta f_1, \Delta f_2, \cdots, \Delta f_n)^{\mathrm{T}},$$

$$\Delta g_1 = (\Delta g_{11}, \Delta g_{12}, \cdots, \Delta g_{1n})^{\mathrm{T}},$$

$$\Delta g_2 = (\Delta g_{21}, \Delta g_{22}, \cdots, \Delta g_{2n})^{\mathrm{T}},$$

$$\Delta g_3 = (\Delta g_{31}, \Delta g_{32}, \cdots, \Delta g_{3n})^{\mathrm{T}},$$

$$\Delta g = (\Delta g_1, \Delta g_2, \Delta g_3).$$

P can be repeatedly calculated with the method of revised feasible direction. The feasible direction $S^{(K)}$ can be solved by

$$S = -(I - \Delta g(\Delta g^{\mathrm{T}} \Delta g)^{-1} \Delta g^{\mathrm{T}}) \Delta f + \Delta g(\Delta g^{\mathrm{T}} \Delta g)^{-1} v \quad (13)$$

where I is a unit matrix, v is a parameter vector. S must be satisfactory with the following formula:

$$\left. \begin{aligned} \Delta f^{\mathrm{T}} S &\leqslant 0 \\ \Delta g^{\mathrm{T}} S &\geqslant 0 \end{aligned} \right\} \quad (14)$$

The pace factor α_K can be determined in the direction S with the linear seeking method. α_K is the maximum of $(1, 2^{-1}, 2^{-2}, \cdots)$, which is satisfactory with

$$f(P^{(K)} + \alpha_K S^{(K)}) - f(P^{(K)}) \leqslant \alpha_K \tau (\Delta f^{(K)})^{\mathrm{T}} S^{(K)}$$

where

$$0 < \tau < 1.$$

3 NON-DERIVATIVE ALGORITHM PROCESS

1) Step 1

To let the repeated number $K = 1$ and give all initial values which are respectively $P^{(1)}$ $\varepsilon^{(1)}$, ($\varepsilon^{(1)} > 0$), β and τ ($\tau \in (0,1)$).

2) Step 2

To calculate $\Delta f(P^{(K)} \varepsilon^{(K)})$ and $\Delta g(P^{(K)} \varepsilon^{(K)})$.

3) Step 3

To calculate the direction $S^{(K)}$ from Eqn.(13). SHI has given the selection of parameter[4]. If $S^{(K)}$ is satisfactory with Eqn.(13), then go to Step 4. Otherwise let $\varepsilon^{(K)} = \varepsilon^{(K)}/2$, then go to Step 2.

4) Step 4

To calculate the pace factor α_K, that is

$$\alpha_K = \max\left\{ \alpha'_K \in (1, 2^{-1}, 2^{-2}, \cdots) \mid f(P^{(K)} + \alpha'_K S^{(K)}) - f(P^{(K)}) \leqslant \alpha'_K \tau (\Delta f^{(K)})^{\mathrm{T}} S^{(K)} \right\} \quad (15)$$

First let $\alpha'_K = 1$, then substitute $\alpha'_K = 1$ into Eqn.(15). If Eqn.(15) is correct, $\alpha_K = 1$. Otherwise let again $\alpha'_K = 1/2$. Repeat the above process. If Eqn.(15) is correct, $\alpha_K = 1/2$. Otherwise repeat the above process until the given α'_K is satisfactory with Eqn.(15) and $\alpha_K = \alpha'_K$.

5) Step 5

To calculate $\Delta_K = f(P^{(K)} + \alpha_K S^{(K)}) - f(P^{(K)})$. If $\Delta_K \leqslant \beta \varepsilon^{(K)}$ in which β is a real number and $\beta > 0$, then go to Step 6. Otherwise let $\varepsilon^{(K)} = \varepsilon^{(K)}/2$, go to Step 2.

6) Step 6

Let $P^{(K+1)} = P^{(K)} + \alpha_K S^{(K)}$ and $K = K + 1$. Then go to Step 2.

7) Step 7

The above process can give a set of points $\left[\boldsymbol{P}^{(K)}\ \varepsilon^{(K)}\ S^{(K)} \right]$. If $\varepsilon \rightarrow 0$, $\Delta f^{\mathrm{T}} S \rightarrow \nabla f^{\mathrm{T}} S$. If $\varepsilon \rightarrow 0$ and $\Delta f^{\mathrm{T}} S \rightarrow 0$, the final $\boldsymbol{P}$ is the optimal solution.

The nonlinear dynamic optimal design of class two put forward in the paper is a nonlinear analytic algorithm which is strict in theory. In practice, it is simple and effective. It is more convenient than analytic algorithms of derivative nonlinear optimum. It opens up a new way to solve the nonlinear dynamic optimal design.

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