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Efficient approach for dynamic simulation of complex fluid networks in frequency domain①

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[Abstract] The conventional transfer matrix models of fluid elements were modified and a convenient method of dealing with junction boundary conditions was introduced. A large-scale fluid network was modeled by standard procedures, and a network was expressed with characteristic matrix and boundary condition matrix. By simple operation of matrix, the dynamic characteristics of a large-scale fluid network was simulated in frequency domain. Validation test on a large-scale pipeline network showed that the proposed method is accurate and practical.

[Key words] hydraulic network model; hydraulic simulation; transfer matrix; large-scale system
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1 INTRODUCTION

The mathematical modeling and dynamic analysis of fluid transmission networks have been the interesting research topic for about 50 years. The wide variety of applications of fluid networks, such as fluid power control, fluid signal processing, petroleum transmission, hydraulic distribution and blood flow, require easy modeling and simulation techniques.

Up to now, the dynamic modeling and analysis of fluid pipeline networks have been studied with the method of impedance[1,2], transmission line theory[3], bond graph modeling[4,5], model approximation[6], infinite products[7] and characteristics[8,9]. The impedance method which is based on transfer matrix models of fluid elements is applied widely in series systems, but is inefficient in modeling of large-scale fluid networks because of complexity of junction boundary conditions. In order to simplify the junction boundary conditions, the transfer matrix models of fluid elements were modified, and the divergent junction and convergent junction were unified, so that the transfer matrix models are always in harmony with junction boundary conditions. Based on the modified models of fluid elements, and unified junction boundary conditions, an efficient approach for modeling and simulation of frequency characteristics of large-scale fluid networks is proposed. To illustrate this method, an example network is modeled and simulated.

2 FLUID ELEMENT TRANSFER MATRIX MODEL

Fig.1(a) shows the conventional transfer matrix model of fluid element. The inward and outward direction is defined as the flow direction at input and output respectively. The establishment of conventional transfer matrix models of fluid elements such as pipeline, T-filter and accumulator are discussed in Ref.[2]. The model is represented as

$$\begin{bmatrix} Q_o \\ P_o \end{bmatrix} = [A] \begin{bmatrix} Q_i \\ P_i \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} Q_i \\ P_i \end{bmatrix} \quad (1)$$

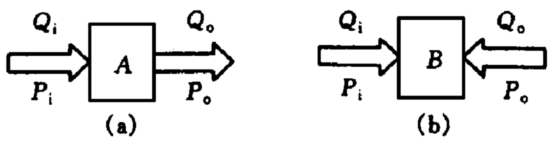

Fig.1 Fluid element model
(a) —Conventional transfer matrix model;
(b) —Modified transfer matrix model

To complex fluid networks with multiple branches, if conventional transfer matrix models were adopted, the constraints of flow direction at two ends of elements would lead to conflict of flow direction in junctions, and the matched equations may be unavailable. To avoid conflict of flow direction in junctions, the flow direction at both input and output were defined as inward direction (Fig.1(b)), so the transfer matrix model is modified as

$$\begin{bmatrix} Q_o \\ P_o \end{bmatrix} = [B] \begin{bmatrix} Q_i \\ P_i \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} Q_i \\ P_i \end{bmatrix} \quad (2)$$

where $B_{11} = -A_{11}$; $B_{12} = -A_{12}$; $B_{21} = A_{21}$; $B_{22} = A_{22}$.

In the modified transfer matrix model, the flow

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direction at two ends is not real flow direction. The four elements of transfer matrix only indicate the value relationships of flow and pressure between two ends. To fluid pipeline, it can be proved that: $[B] = [B]^{-1}$. It is the property of modified transfer matrix model of the fluid pipeline and other symmetrical elements.

3 BOUNDARY CONDITIONS

3.1 Boundary conditions of elements in series

It is showed in Fig.2 that n elements are in series. The modified transfer matrix of each element is indicated by $[B_i]$. The boundary conditions between two elements are: $P_o^{(i-1)} = P_i^{(i)}$, $Q_0^{(i-1)} = -Q_i^{(i)}$. Thus the equivalent transfer matrix is obtained by

$$[B] = [B_n][K][B_{n-1}][K]\cdots[B_2][K][B_1] \quad (3)$$

where $[K] = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$.

Fig.2 Elements in series

Unlike the conventional transfer matrix model, the equivalent transfer matrix was obtained not only by multiplication of modified transfer matrix of elements, but also by the matrix K connected between the two adjoining elements.

3.2 Junction boundary conditions

Figs.3(a) and (b) are examples of arrangements of divergent and convergent junction with two branches. When the branch number is more than 3, the situation will be more complex. In "Tree" type systems, only divergent junctions are involved, the flow direction at each branch is definite, so the equivalent transfer matrix in main line at junctions can be obtained with the chain rule[2], and the overall equivalent transfer matrix of network can be established easily. But if the network involves divergent and convergent junctions at the same time, it may be difficult to acquire the matched network equations by means of conventional transfer matrix and junction boundary conditions. In this paper, no matter what type the junction is and whether real flow direction of branches are, the outward direction is defined as flow direction in each branch at the junction (Fig.3(c)), so that the junction boundary conditions in network are always matched with branch transfer matrix models. The boundary condition in Fig.3(c) can be expressed in matrix form:

Pressure boundary condition: $\boldsymbol{EP} = 0$. (4)

Flow boundary condition: $\boldsymbol{DQ} = 0$. (5)

where

$$\boldsymbol{E} = \begin{bmatrix} 1 & -1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & -1 & 0 & \cdots & 0 & 0 \\ \vdots & & & \vdots & & & \vdots \\ 0 & 0 & 0 & 0 & \cdots & -1 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 1 & -1 \end{bmatrix} \in \boldsymbol{R}^{(n-1)\times n}$$

$$\boldsymbol{D} = [1 \quad 1 \quad 1 \quad \cdots \quad 1 \quad 1] \in \boldsymbol{R}^{1\times n}$$

$$\boldsymbol{P} = [P_1 \quad P_2 \quad P_3 \quad \cdots \quad P_{n-1} \quad P_n]^{\mathrm{T}}$$

$$\boldsymbol{Q} = [Q_1 \quad Q_2 \quad Q_3 \quad \cdots \quad Q_{n-1} \quad Q_n]^{\mathrm{T}}$$

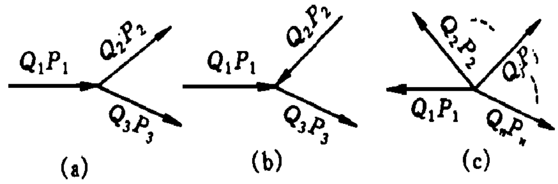

Fig.3 Junction boundary condition
(a) —Divergent junction; (b) —Convergent junction; (c) —Unified junction

3.3 Terminal boundary conditions

Terminal boundary conditions are usually expressed in form of impedance Z_L or admittance G_L.

$$\frac{P_L}{Q_L} = Z_L = \frac{1}{G_L} \quad (6)$$

when $G_L = 0$, $P_L = 0$, the end is open.

When $Z_L = \infty$, $Q_L = 0$, the end is closed. In the dynamic analysis of fluid networks, it is usual to take the closed end as termination.

4 NETWORK MODELING AND SIMULATION

Fig.4 illustrates the structure of the fluid network in consideration. The flow disturbance q_r is exerted to network at the point a. In the figure, i and o are used to represent two ends of each branch respectively, but not indicate the real flow direction. The equivalent modified transfer matrix of branch j is $[B^{(j)}]$. So

Fig.4 Scheme of complex fluid network

$$\begin{bmatrix} Q_o^{(j)} \\ P_o^{(j)} \end{bmatrix} = \begin{bmatrix} B_{11}^{(j)} & B_{12}^{(j)} \\ B_{21}^{(j)} & B_{22}^{(j)} \end{bmatrix} \begin{bmatrix} Q_i^{(j)} \\ P_i^{(j)} \end{bmatrix} \quad (7)$$

It can be transferred as

$$\begin{bmatrix} P_i^{(j)} \\ P_o^{(j)} \end{bmatrix} = \begin{bmatrix} T_{11}^{(j)} & T_{12}^{(j)} \\ T_{21}^{(j)} & T_{22}^{(j)} \end{bmatrix} \begin{bmatrix} Q_i^{(j)} \\ Q_o^{(j)} \end{bmatrix} \quad (8)$$

where $T_{11}^{(j)} = -\dfrac{B_{11}^{(j)}}{B_{12}^{(j)}}$; $T_{12}^{(j)} = \dfrac{1}{B_{12}^{(j)}}$; $T_{21}^{(j)} = -\dfrac{|B^{(j)}|}{B_{12}^{(j)}}$; $T_{22}^{(j)} = \dfrac{B_{22}^{(j)}}{B_{12}^{(j)}}$; $j = 1, 2, 3, \cdots, 9$.

For convenient, define

$$\boldsymbol{P} = [\, P_i^{(1)} \quad \overline{\boldsymbol{P}} \,]^T \in \boldsymbol{C}^{18\times 1}$$

$$\overline{\boldsymbol{P}} = [\, P_o^{(1)} \quad P_i^{(2)} \quad P_o^{(2)} \quad \cdots \quad P_i^{(j)} \quad P_o^{(j)} \quad \cdots \quad P_i^{(9)} \quad P_o^{(9)} \,]^T \in \boldsymbol{C}^{17\times 1}$$

$$\boldsymbol{Q} = [\, Q_i^{(1)},\ Q_o^{(1)},\ Q_i^{(2)},\ Q_o^{(2)},\ \cdots,\ Q_i^{(j)},\ Q_o^{(j)},\ \cdots,\ Q_i^{(9)},\ Q_o^{(9)} \,]^T \in \boldsymbol{C}^{18\times 1}$$

From Eqn.(8), the overall network characteristic equation is available:

$$\boldsymbol{P} = \boldsymbol{T}\boldsymbol{Q} \qquad (\boldsymbol{T} \in \boldsymbol{C}^{18\times 18}) \quad (9)$$

where

$$\boldsymbol{T} = \begin{bmatrix} T_{11}^{(1)} & T_{12}^{(1)} & & & & \\ T_{21}^{(1)} & T_{22}^{(1)} & & & & \\ & & T_{11}^{(j)} & T_{12}^{(j)} & & \\ & & T_{21}^{(j)} & T_{22}^{(j)} & & \\ & & & & T_{11}^{(9)} & T_{12}^{(9)} \\ & & & & T_{21}^{(9)} & T_{22}^{(9)} \end{bmatrix}$$

Define $\boldsymbol{U} = \{ T_{1j} \} \in \boldsymbol{C}^{1\times 18}$ ($j = 1, 2, \cdots, 18$); $\boldsymbol{V} = \{ T_{ij} \} \in \boldsymbol{C}^{17\times 18}$ ($i = 2, \cdots, 18$; $j = 1, 2, \cdots, 18$).

Thus

$$P_i^{(1)} = \boldsymbol{U}\boldsymbol{Q} \quad (10)$$

$$\overline{\boldsymbol{P}} = \boldsymbol{V}\boldsymbol{Q} \quad (11)$$

The flow boundary condition of junctions and terminations can be obtained as

$$\boldsymbol{D}\boldsymbol{Q} = \boldsymbol{R} \qquad (\boldsymbol{D} \in \boldsymbol{R}^{9\times 18}) \quad (12)$$

where

$$\boldsymbol{D} = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&1&1&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&1&1&0&0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&1&1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&0&1&0 \\ 0&0&0&0&0&0&0&1&1&0&0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0 \end{bmatrix}$$

$\boldsymbol{R} = [\, q_r, 0, \cdots, 0 \,]^T \in \boldsymbol{C}^{9\times 1}$.

The pressure boundary condition of junctions can be represented as

$$\boldsymbol{E}\overline{\boldsymbol{P}} = 0 \qquad (\boldsymbol{E} \in \boldsymbol{R}^{9\times 17}) \quad (13)$$

where

$$\boldsymbol{E} = \begin{bmatrix} 1&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&-1&0&0&0&0&0&0&0 \\ 0&0&1&-1&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&-1&0&0&0&0&0 \\ 0&0&0&0&1&-1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&1&-1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&-1&0 \\ 0&0&0&0&0&0&1&-1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&-1 \end{bmatrix}$$

From Eqns.(11) ~ (13), it can be obtained that:

$$W\boldsymbol{Q} = \boldsymbol{Y} \quad (14)$$

where $W = \begin{bmatrix} D \\ EV \end{bmatrix} \in \boldsymbol{C}^{18\times 18}$, $\boldsymbol{Y} = [\, q_r, 0, \cdots, 0 \,]^T \in \boldsymbol{C}^{18\times 1}$.

From Eqns.(9) and (14), it can be derived that

$$\boldsymbol{P} = \boldsymbol{Z}\boldsymbol{Y} \quad (15)$$

where $\boldsymbol{Z} = \boldsymbol{T} W^{-1}$

Define $\{ Z_{j,1} \} \in \boldsymbol{C}^{18\times 1}$ ($j = 1, 2, \cdots, 18$), thus

$$\boldsymbol{P} = \overline{\boldsymbol{Z}} q_r \quad (16)$$

So

$$\boldsymbol{P}_m = \overline{\boldsymbol{Z}}_m q_r \ (m = 1, 2, \cdots, 18) \quad (17)$$

The transfer functions of pressure at any location to input flow disturbance are obtained in dimensionless G_P.

$$G_P = \frac{P_m}{z_{c1}\, q_r} = \frac{\overline{Z}_m}{z_{c1}} \quad (18)$$

where $z_{c1} = \dfrac{\rho c}{a}$ is characteristic impedance of pipeline, ρ is fluid density, c is sonic speed in the fluid, a is cross-section area of pipeline.

The Eqn.(18) is the base of simulation of dynamic characteristics of the network.

5 EXAMPLE OF MODELLING

To prove the efficiency and accuracy of proposed modeling method, the experiment curves of Ref.[10] are used to compare with the calculating results. The network scheme and parameters are illustrated in Fig.4 and Table 1. Fig.5 and Fig.6 are the results of experiment and calculation respectively, in which the horizontal coordinate is the dimensionless value lw/c, where $l = l_1 + l_2 + l_3$. In Fig.6, the curve indicated by " * " is the transfer function of a, which represents the impedance characteristics of the network, and the main resonant frequency is 59 Hz($lw/c = 3.5958$). The curves indicated by real line and dot line are the transfer functions of c and d individually, in which the magnitude are considerable small in the vicinity of main resonant frequency compared with the transfer function a, but in low frequency the results are very close. This is because that the induced pressure pulsation of higher frequency propagated in the network is filtered out by pipelines. It is clear that the calculating results are good agreement

Table 1 Parameters of network for calculation[10]

l_i/cm	519	264	523	256	158	625	279	137	518
r_i/cm	0.46	0.46	0.33	0.82	0.62	0.62	0.62	0.62	0.46

Parameters of fluid are $\nu = 1.05 \times 10^{-4}\ m^2/s$, $\rho = 862\ kg/m^3$, $c = 1\ 350\ m/s$.

Fig.5 Experimental results

Fig.6 Calculated results

with that of the experiment, so the proposed method is reliable and accurate.

6 CONCLUSIONS

1) Compared to conventional transfer matrix model, the modified model breaks down the constraints of flow direction at two ends of branches, and always matches with any junction boundary conditions.

2) The paper puts the divergent and convergent junction into a unified form, and provides a unified method to express the junction boundary conditions. To describe the scheme of a network, it is only necessary to describe the topology structure of the network, need not to acquire the details in junctions, so it leads to simplification.

3) The proposed modeling approach mainly embraces the characteristic matrix, pressure and flow boundary condition matrix of network, in which the value of elements is acquired by standard procedures, so the method is very convenient for engineering applications.

4) The proposed method only involves in simple matrix manipulation, and the calculating accuracy is assured. To elements of transfer matrix, if they can be expressed in frequency domain, it is unnecessary to be simplified. So the method shows superiority in dealing with distributed parameter systems.

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