

## Complicated motion of particles on vibration screen surface<sup>①</sup>

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**Abstract:** The particles' motion on screen surface was studied by the way of nonlinear dynamics. It was found that the particles' motion may be changed from one form to another with its vibration strength. When the vibration strength  $K$  is bigger than 1 and smaller than 1.33, the particles' motion is the bifurcating type; when  $K$  is bigger than 1.33 and smaller than 1.67, its motion becomes the double bifurcation type; when  $K$  is bigger than 1.67, its motion changes into chaos motion type. On basis of studying all effects of large vibration strength on particles penetrating, it was pointed out that a larger vibration strength  $K$  of screen surface will increase the strength of particles' motion, and then increase their probability of penetrating screen surface. A primary theory of particles' motion under large vibration strength for moisture fine coal was established.

**Key words:** screen surface; vibration strength; bifurcation and chaos

**Document code:** A

### 1 INTRODUCTION

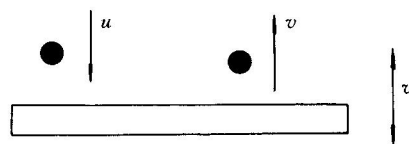
Today, the motion theory of particles on screen surface is an important research direction of screening technology and theory<sup>[1~8]</sup>. By means of studying motion regularity of particles, we can obtain the optimum kinetic parameters for the industrial screening and raise its efficiency.

The material being screened has variable shape and size, with different surface moisture and clay minerals contents. The motion of particles is affected by displacement, velocity, acceleration of screen surface and friction among particles. So the particles' motion on screen surface is very complicated. So far, there are two kinds of kinetic theory of particles moving on screen surface, one is the linear and constant theory, and the other is the random theory. On basis of single, linear and constant theory, the  $K$  (vibration strength coefficient of screen machine) must be smaller than or equal to 3.3<sup>[9~10]</sup>. In fact, the  $K$  must be bigger than 3.3 for screen machines when it deals with high moisture fine ma-

terials<sup>[5, 11, 12]</sup>. The random theory is difficult to decide the design parameters of the screen machine. In this work, we studied the complicated motion of particles on screen surface and have developed one new screening theory.

### 2 NONLINEAR MOTION THEORY OF PARTICLES ON SCREEN SURFACE

The particles' kinetic motion regularity on screen surface was governed by the way of screen surface kinetics. The screen surface kinetic regularity was decided by screen vibration amplitude and frequency. As shown in Fig.1, a ball falls



**Fig.1** Conceptual diagram of collision and rebound speed of small ball on screen surface

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freely on a screen surface. When the ball runs to the screen surface and its velocity is  $u$ . The amplitude and frequency of screen surface's vibration are  $A$  and  $\omega$  respectively. The kinetic velocity of screen surface is  $w = A\omega \cos \omega t$ . The velocity of ball bouncing back is  $v$  and the time of ball touched the screen surface in number  $i$  is  $t_i$ . The bouncing equation is as following (Holms model)<sup>[13, 14]</sup>:

$$v(t_i) - w(t_i) = -\alpha[u(t_i) - w(t_i)] \quad (1)$$

where  $\alpha$  is the velocity resumption constant.

At the time  $t_{i+1}$ , the velocity of ball to touch screen surface is

$$\begin{aligned} u(t_{i+1}) &= -v(t_i) \\ \text{so } v(t_{i+1}) - w(t_{i+1}) &= -\alpha[-v(t_i) - w(t_{i+1})] \end{aligned} \quad (2)$$

The time of impacting can be omitted. So, we have

$$t_{i+1} - t_i = 2v(t_i)/g \quad (3)$$

From Eqns (2) and (3), we can obtain two dimensional mapping:

$$\begin{cases} x_{i+1} = x_i + y_i \\ y_{i+1} = \alpha y_i + r \cos(x_i + y_i) \end{cases} \quad (4)$$

where  $x = \omega t$ ;  $y = 2\omega v/g$ ;

$$r = 2\omega^2(1 + \alpha) \quad A/g = 2K(1 + \alpha),$$

$$K = A\omega^2/g$$

There is a static point  $O(\pi/2, 0)$  in Eqn. (4). The stability of point  $O$  can obtain from Jacobi matrix of Eqn. (4). The Jacobi matrix is as following:

$$\begin{aligned} J &= \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{bmatrix} \\ &= \begin{bmatrix} \frac{\partial x_{n+1}}{\partial x_n} & \frac{\partial x_{n+1}}{\partial y_n} \\ \frac{\partial y_{n+1}}{\partial x_n} & \frac{\partial y_{n+1}}{\partial y_n} \end{bmatrix} \left( \frac{\pi}{2}, 0 \right) \\ &= \begin{pmatrix} 1 & 1 \\ -r & \alpha - r \end{pmatrix} \end{aligned} \quad (5)$$

The eigen values of Eqn. (5) are

$$|\lambda E - J| = \begin{vmatrix} \lambda - 1 & -1 \\ r & \lambda + r - \alpha \end{vmatrix} = 0$$

$$\lambda^2 + \lambda(r - \alpha - 1) + \alpha = 0$$

$$\lambda_{1,2} = [(1 + \alpha - r) \pm$$

$\sqrt{(r - \alpha - 1)^2 - 4\alpha}]/2$   
where  $|\lambda_1|, |\lambda_2|$  are all smaller than 1. The static point  $O$  is stable. According to linear constant theory, let  $\alpha = 0.5$ , the condition of stability of static point  $O$  is  $r < 3$ , namely,  $K < 1$ . When  $r \geq 3$ , one of the  $|\lambda_1|$  and  $|\lambda_2|$  must be bigger than 1. The static point  $O$  is unstable. We iterate Eqn. (4) in order to obtain numerical result. The last ten results are as listed in Table 1.

We can learn from Table 1 that when  $r < 3$ , there is a stable zero velocity; when  $3 < r \leq 4$ , there are two stable velocities, one active and the other negative; when  $4 < r < 5$ , there are four stable velocities, two active and two negative; when  $r > 5$ , there is no stable velocity.

### 3 THEORY ANALYSIS OF KINETIC REGULARITY OF PARTICLES ON SCREEN SURFACE

Based on the iterated results of Eqn. (4), we drew the phase figure as shown in Fig. 2. The coordinate  $x$  is  $A\omega^2/g$  and the coordinate  $y$  is  $2\omega v/g$ . We can learn from Fig. 2 that when  $K < 1$ , there is a stable static point,  $y = 0$ . It shows that the particles are relatively static on screen surface. In fact, the particles can't jump. When  $1 < K \leq 1.33$  ( $3 < r \leq 4$ ), there are two stable velocities, one active and the other negative. The negative velocity is omitted. In this case, the particles will jump periodically. It

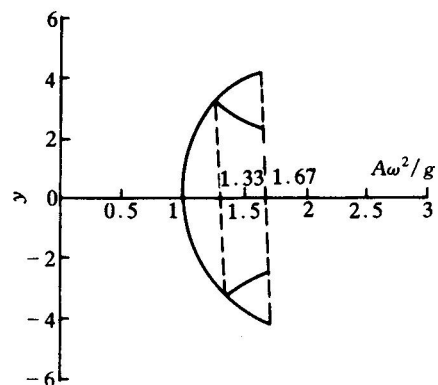
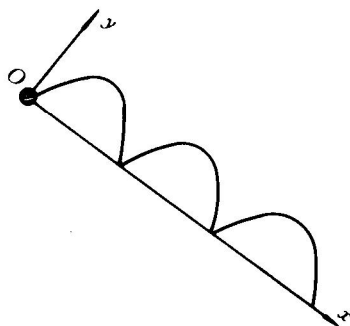
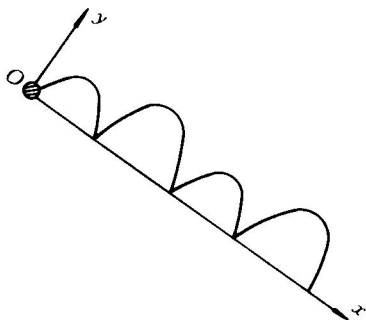
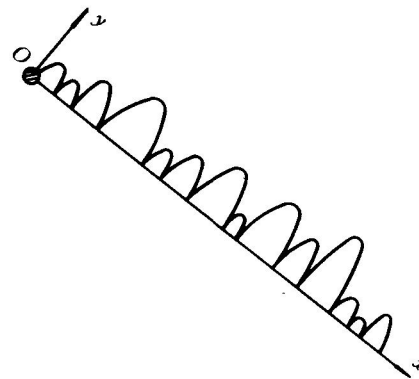


Fig. 2 Bifurcation diagram of particles' motion

**Table 1** Iterative results with different value of  $r$ 

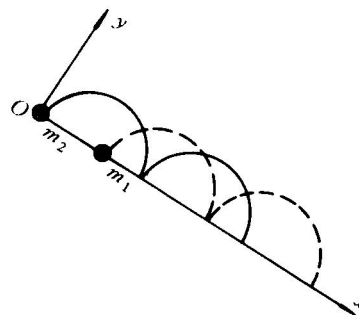
$r$	$y_1$	$y_2$	$y_3$	$y_4$	$y_5$	$y_6$	$y_7$	$y_8$	$y_9$	$y_{10}$
3	$2 \times 10^{-6}$	$2 \times 10^{-6}$	$2 \times 10^{-6}$	$2 \times 10^{-6}$	$2 \times 10^{-6}$	$2 \times 10^{-6}$	$2 \times 10^{-6}$	$2 \times 10^{-6}$	$2 \times 10^{-6}$	0
4	2.551	-2.551	2.551	-2.551	2.551	-2.551	2.551	-2.551	2.551	-2.551
5	3.221	-3.056	3.082	-3.247	3.221	-3.056	3.082	-3.247	3.221	-3.056
6	7.269	2.665	-0.521	-4.711	1.676	7.340	9.081	-1.411	-2.164	4.571
10	1.021	6.942	6.279	-4.328	7.097	12.650	15.067	-2.365	6.913	6.537

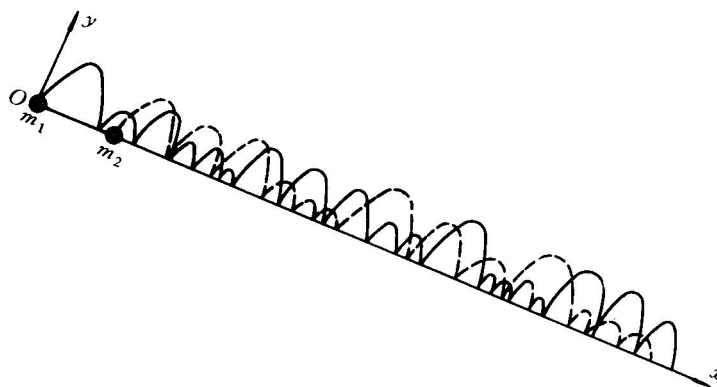
is called periodical bifurcation motion in nonlinear dynamics, its moving trace as shown in Fig. 3. When  $1.33 < K \leq 1.67$  ( $4 < r \leq 5$ ), there are four stable velocities, two active and two negative respectively. The negative velocities are omitted. The particles will jump on screen surface with double periodical bifurcation, its trace as shown in Fig. 4. When  $K > 1.67$  ( $r > 5$ ) there is no stable velocity in many iterations. The particles will not jump periodically. That is called chaos motion, its trace as shown in Fig. 5.

**Fig.3** Conceptual diagram of periodic bifurcation motion of particles**Fig.4** Conceptual diagram of multi-periodic bifurcation motion of particles**Fig.5** Conceptual diagram of particles' chaotic motion

#### 4 PARTICLES' BIFURCATION AND CHAOS MOTION AFFECTING PARTICLES PENE-TRATING

When  $K \leq 1$ , the particles move with the screen deck. When  $1 < K \leq 1.67$ , the particles will jump on screen surface periodically. If there are two particles with the masses of  $m_1$  and  $m_2$ , they will fall on the screen surface one after another as shown in Fig. 6. Their turns of jumping and probabilities of penetrating are constant. As shown in Fig. 7, when  $K > 1.67$ , the two parti-

**Fig.6** Conceptual diagram of two particles' motion on screen surface with  $1 < K \leq 1.67$



**Fig.7** Conceptual diagram of two particles' motion on screen surface with  $K > 1.67$

cles will fall on the screen surface one after another. Because of chaos motion, the motion order of the former and the latter changes. This phenomenon will result in the particles impacting, stopping and backing. So, it increased the probability of penetrating.

From the theory analysis, we know that the large acceleration of vibration screen surface will increase probability of penetrating. This is a new development screen theory and has been tested by screen machine for moist fine materials. For example, the acceleration of flip-flow screen is over 50 g.

## 5 CONCLUSIONS

In this paper, we analyzed the complicated kinetic motion regularity of particles on screen surface from nonlinear dynamics and developed linear and constant kinetic theory. Following conclusions are obtained:

(1) When  $K < 1$ , the particles will be static on the screen surface.

(2) When  $1 < K \leq 1.33$ , the particles will result in periodical bifurcation.

(3) When  $1.33 < K \leq 1.67$ , the particles will result in double periodical bifurcation.

(4) When  $K > 1.67$ , the particles will result in chaos motion on screen surface.

(5) Large vibration strength will increase the probability of penetrating.

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