A CONTROVERTIBLE EQUATION IN "VIBRATION PROBLEMS IN ENGINEERING" BY TIMOSHENKO S[①]

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ABSTRACT A discussed problem has been proposed, which is on a famous book, "Vibration Problems in Engineering", written by Timoshenko S. The problem is that the equation (equation 1.77e, p.135) in original Chapter 1 passage 15 is wrong. A revised conclusion in theory has been inferred tightly. After dimension analyzing and study on exercise answer, the equation's dimension has been proved to be inconsistent and not a printing mistake. The revised conclusion has theoretical meaning in vibration and practical significance in engineering computing.

Key words vibation problems engineering controvertible problem

1 INTRODUCTION

"VIBRATION PROBLEMS IN ENGINEERING (4th Ed)", written by Timoshenko S, an American mechanics professor, is a famous work about vibration engineering and plays an important role in the research of vibration theory, engineering calculation and education. The book is clear, compact, coherent and reflects the application tendency of calculation technique in vibration engineering. Readers may be impressed by its tightness in theory and the accuracy in calculation. However, in the course of studying this book, we found out there was a possible mistake in equation (1.77e) of Chapter 1 passage 15. In order to obtain an accurate conclusion, the possible mistake will be discussed in detail.

A numerical method about one-degree system subjected to an arbitrary force is discussed in original Chapter 1 Passage 15, whose mechanics model is illustrated in Fig. 1.54, p130. For good accuracy, a solving process utilizing piecewise linear interpolation forcing function imitate the forcing function in the original (see also Fig. 1.57, p134 in original). As for undamped and forced vibration, the response X_i of the system at time $t = t_i$ can be calculated in equation (1.77e) in the original.

$$\begin{aligned} X_i = \; & x_0 \cos p t_i + \frac{\dot{x}_0}{p} \sin p t_i + \\ & \frac{1}{k} \sum_{j=1}^{i} \{ Q_{j-1} [\cos p (t_i - t_j) - \\ & \cos p (t_i - t_{j-1})] + \\ & \frac{\Delta Q_j}{k \Delta t_j} [\Delta t_j \cos p (t_i - t_j) + \\ & \frac{1}{p} \sin p (t_i - t_j) - \\ & \frac{1}{p} \sin p (t_i - t_{j-1})] \} \end{aligned} \quad (1)$$

We think there is a mistake in this equation and testify it as follows.

2 DEDUCTION

Referring to original Fig. 1.57, at the moment $t = t_i$, the response X_i shoud be equal to the free response X_0 stimulated by the initial

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conditions plus the forced response x_i stimulted by a series of linear forcing functions Q_1, Q_2, $\cdots$, Q_i.

The free response X_0 stimulated by the initial conditions can be calculated with the following equation:

$$X_0 = x_0 \cos p t_i + \frac{\dot{x}_0}{p} \sin p t_i$$

As for forced response x_i, the Duhamel integral used by the original is still adopted. As for undamped one-degree system stimulated by an arbitrary force function, $Q = F(t')$, its response may be expressed as

$$x = \frac{1}{p}\int_0^t q \sin[p(t - t')]\mathrm{d}t'$$

where t is a constant, t' is a variable, $q = Q/m$.

Referring to the original Fig. 1. 57, the piecewise linear interpolation forcing function Q_i is

$$Q_j = \frac{\Delta Q_j}{\Delta t_j} + t'\left(Q_{j-1} - \frac{\Delta Q_j}{\Delta t_j} t_{j-1}\right)$$

where

$$\Delta Q_j = Q_j - Q_{j-1};\ \Delta t_j = t_j - t_{j-1}$$

Assuming

$$q_j = Q_j / m (j = 1, 2, \cdots, i)$$

according to the Duhamel integral equation

$$\begin{aligned} x_i &= \frac{1}{p}\int_{t_0}^{t_i} q \sin p(t_i - t')\mathrm{d}t' \\ &= \frac{1}{p}\Big[\int_{t_0}^{t_1} q_1 \sin p(t_i - t')\mathrm{d}t' + \\ &\quad \int_{t_1}^{t_2} q_2 \sin p(t_i - t')\mathrm{d}t' + \\ &\quad \cdots + \int_{t_{i-1}}^{t_i} q_i \sin p(t_i - t')\mathrm{d}t'\Big] \\ &= \frac{1}{p}\sum_{j=1}^{i}\int_{t_{j-1}}^{t_j} q_j \sin p(t_i - t')\mathrm{d}t' \\ &= \frac{1}{p}\sum_{j=1}^{i}\Big[\int_{t_{j-1}}^{t_j} \frac{Q_{j-1}}{m}\sin p(t_i - t')\mathrm{d}t' + \\ &\quad \int_{t_{j-1}}^{t_j} \frac{\Delta Q_j}{m\,\Delta t_j} t' \sin p(t_i - t')\mathrm{d}t' - \\ &\quad \int_{t_{j-1}}^{t_j} \frac{\Delta Q_j}{m\,\Delta t_j} t_j \sin p(t_i - t')\mathrm{d}t'\Big] \end{aligned}$$

integrating this equation yields

$$\begin{aligned} x_i = \frac{1}{k}\sum_{j=1}^{i}\{ & Q_{j-1}[\cos p(t_i - t_j) - \\ & \cos p(t_i - t_{j-1})] + \\ & \frac{\Delta Q_j}{\Delta t_j}[\Delta t_j \cos p(t_i - t_{j-1}) + \\ & \frac{1}{p}\sin p(t_i - t_j) - \\ & \frac{1}{p}\sin p(t_i - t_{j-1})]\} \end{aligned}$$

thus the response X_i at time $t = t_i$ is

$$\begin{aligned} X_i = {} & x_0 \cos p t_i + \frac{\dot{x}_0}{p}\sin p t_i + \\ & \frac{1}{k}\sum_{j=1}^{i}\{Q_{j-1}[\cos p(t_i - t_j) - \\ & \cos p(t_i - t_{j-1})] + \\ & \frac{\Delta Q_j}{\Delta t_j}[\Delta t_j \cos p(t_i - t_{j-1}) + \\ & \frac{1}{p}\sin p(t_i - t_j) - \\ & \frac{1}{p}\sin p(t_i - t_{j-1})]\} \end{aligned} \tag{2}$$

3 DISCUSSION AND CONCLUSIONS

Comparing Eqn. (2) with Eqn. (1), it is known that the coefficient of the second part of the forced response is $\Delta Q_j / t_j$ or $\Delta Q_j / (k\,\Delta t_j)$ respectively in the two equations. Which is correct? It can be judged by analyzing the dimension. The unit of displacement X_i is meter, that of force Q is Newton, then the unit of the second part of Eqn. (1) must be meter. Calculating the dimension of $\frac{1}{k}\frac{\Delta Q_j}{k\,\Delta t_j}\Delta t_j \cos p(t_i - t_j)$ yields that its unit is $\mathrm{m^2/N}$. So the second part of Eqn. (1) is proved obviously wrong. Calculating the dimension of second part according to Eqn. (2), $\frac{1}{k}\frac{\Delta Q_j}{\Delta t_j}\Delta t_j \cos p(t_i - t_j)$, yields that its unit is meter. Thus it can be seen that Eqn. (2) is correct and Eqn. (1) is wrong.

The mistake in Eqn. (1) will cause large error of the result. Because spring constant k in engineering construction ususally is very big, such as the original points out, k has 3 or 4 figures, maybe even larger than that. The calculating error can reach about 30% caused due to the mistake of Eqn. (1) through computing result.

For example, the original exercise 1. 15−

2, p152, requires to determine the expression $\dot{X}_i/p$ of Eqn. (1), the original answer, p142, is

$$\frac{\dot{X}_i}{p} = - x_0 \sin p t_i + \frac{\dot{x}_0}{p} \cos p t_i + \frac{1}{k} \sum_{j=1}^{i} \{ Q_{j-1} [- \sin p(t_i - t_j) + \sin p(t_i - t_{j-1})] + \frac{\Delta Q_j}{k \Delta t_j} [- \Delta t_j \sin p(t_i - t_j) + \frac{1}{p} \cos p(t_i - t_j) - \frac{1}{p} \cos p(t_i - t_{j-1})] \}$$

Comparing the original equation with this expression shows that this expression makes the same mistake as Eqn. (1). This means that the mistake isn' t a printing mistake. The correct answer is given as

$$\frac{\dot{X}_i}{p} = - x_0 \sin p t_i + \frac{\dot{x}_0}{p} \cos p t_i + \frac{1}{k} \sum_{j=1}^{i} \{ Q_{j-1} [- \sin p(t_i - t_j) + \sin p(t_i - t_{j-1})] + \frac{\Delta Q_j}{\Delta t_j} [- \Delta t_j \sin p(t_i - t_{j-1}) + \frac{1}{p} \cos p(t_i - t_{j-1}) - \frac{1}{p} \cos p(t_i - t_{j-1})] \}$$

Through the discussion above, it is believed that the orignal equation 1.77e is wrong, Eqn. (2) deduced in this paper is correct.

REFERENCE

1 Timosheko S, Young D H and Weaver W. Vibration Problems in Engineering, Fourth Edition. California: California University Press, 1974: 129– 142.

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