CRITERION AND APPRAISABLE FUNCTION OF SIZE CHARACTERISTICS①

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ABSTRACT The concepts of both criterion and appraisable function of size characteristics are first proposed. The size character of a material can be simply judged using the criterion, and its size distribution can be accurately appraised by use of the appraisable function. The criterion $D = (a_1/a_2)^{[1/(b_2-b_1)]}$ and the appraisable function $J(a, b) = \lg a + b\lg x$ are deduced from theories and have been examined with size analysis data, the results of which are satisfactory.

Key words size size characteristics criterion appraisable function

1 INTRODUCTION

Size is a physical measure of geometric dimension of material particles. In comminution engineering, the size character of mixture material plays a very important part in determining the scheme and flowsheet of comminution. Meanwhile, the physical and the chemical natures of a material depend to a great extent on its size character in such industries as metallurgy, building material, chemistry, and coal, etc[1,2]. General method to describe the size character is to use expressions of size distribution of particles. There are four kinds of common expressions, among which the Rosin-Rammler-Bennet expression is most widely used[3]:

$$F(x) = 1 - \exp(-ax^b), \quad x \in (0, 1) \qquad (1)$$

where x is the relative size of mineral particles, $x = d/d_{\max}$ (d is the size of mineral particles and $d_{\max}$ is the maximum size); $F(x)$ is the cumulative mass percentage undersize; both a and b are parameters.

It can be known from equation (1) that the size character of given mineral particles depends completely on parameters a and b[4]. For this reason parameters a and b were defined by the first author of this paper as "the characteristic parameters of size distribution" and the determining method was mentioned[5,6]. Evidently, the characteristic parameters a and b represent directly the size character of mineral particles. It is thus clear that when b is a constant, $F(x)$ increases with a's growth, i. e. the fraction of fine size range of the material increases with increasing a, see Fig. 1 (a). When a is a constant, $F(x)$ decreases with b's growth, i. e. the fraction of fine size range of the material decreases with increasing b, see Fig. 1 (b). Obviously, if a increases and b decreases, $F(x)$ will increase. And if a decreases and b increases, $F(x)$ will decrease.

But how does $F(x)$ change when both a and b increase or decrease simultaneously? Which will increase in the material, the fraction of fine size range or that of coarse size range? These problems were studied in this paper. The criterion of size characteristics and the appraisable function of size characteristics were established to describe comprehensively the law of effect of characteristic parameters a and b of size distribution on $F(x)$.

2 DETERMINATION OF THE CRITERION OF SIZE CHARACTERISTICS

Analysis and calculation show that there is a

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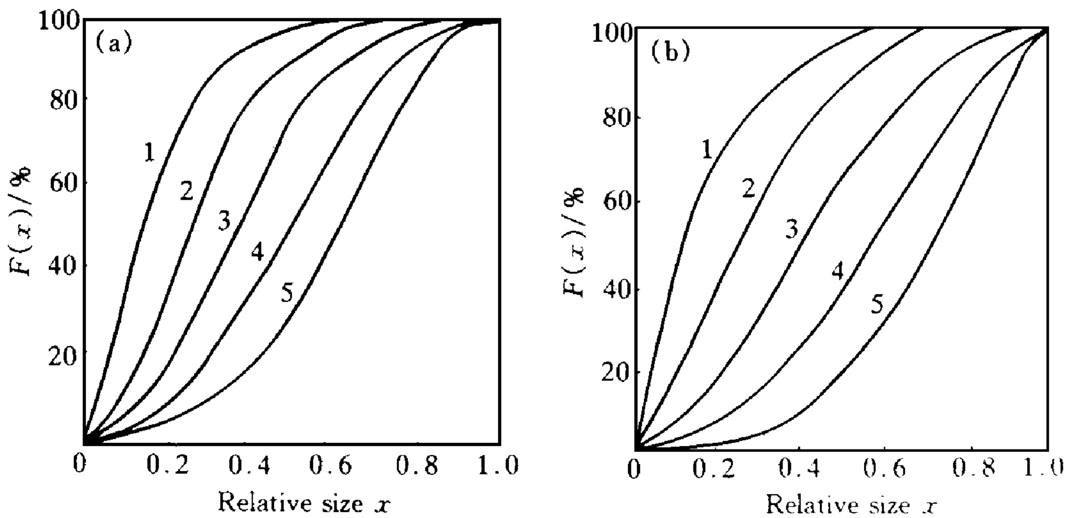

Fig. 1 Effects of parameters *a*, *b* on curves of size characteristics

(a) 1—a= 12; 2—a= 10; 3—a= 8; 4—a= 6; 5—a= 4;

(b) 1—b= 1.5; 2—b= 2.0; 3—b= 2.5; 4—b= 3.0; 5—b= 3.5

crosspoint between $F(x)$ —x curves as both a and b increase or decrease at the same time. Assume there are two groups of materials $M_1(a_1, b_1)$ and $M_2(a_2, b_2)$, and $a_2 > a_1$, $b_2 > b_1$. When a and b are variables and x is a constant, equation (1) can be revised as:

$$F(a, b) = 1 - \exp(-ax^b) \quad (2)$$

The total differential of $F(a, b)$ is:

$$dF = \frac{\partial F}{\partial a}da + \frac{\partial F}{\partial b}db \quad (3)$$

$$dF = x^b \cdot \exp(-ax^b)[da + a\ln(x) \cdot db] \quad (4)$$

Let $dF = 0$, because $x^b \cdot \exp(-ax^b) \neq 0$, then:

$$da + a\ln x\, db = 0$$

$$da/a = -\ln x\, db \quad (5)$$

Integrating equation (5) we get:

$$\int_{a_1}^{a_2} \frac{1}{a}da = -\ln x \int_{b_1}^{b_2} db \quad (6)$$

Thus

$$x = (a_1/a_2)^{[1/(b_2-b_1)]} \quad (7)$$

Let

$$D = (a_1/a_2)^{[1/(b_2-b_1)]} \quad (8)$$

then D is the lateral coordinate of the crosspoint of $F_1(x)$ and $F_2(x)$.

It can be seen from Fig. 2 that, when $x > D$, $F_2(x) > F_1(x)$, and that when $x < D$, $F_2(x) < F_1(x)$.

Assume the relative size $x \in (0, 0.5)$ to be fine particle range and $x \in (0.5, 1)$ to be coarse particle range. Then as $D > 0.5$, the less the values of the characteristic parameters a and b are, the greater the fraction of fine particle size is (shown in Fig. 3(a)); and as $D < 0.5$, the greater the values of the characteristic parameters a and b are, the greater the fraction of fine particle size is (shown in Fig. 3(b)). It is thus clear that the value of D provides objective basis for judging the size distribution characteristics of two groups of given materials. In point of this, D is defined as "the criterion of size characteristics", or "D-criterion" for short, the expression of which is $D = (a_1/a_2)^{[1/(b_2-b_1)]}$.

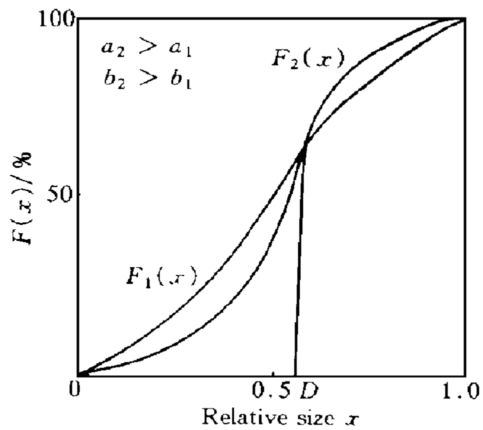

Fig. 2 Curves of *F*(*x*) —*x*

Practically, if the characteristic parameters of size distribution of two groups of materials are

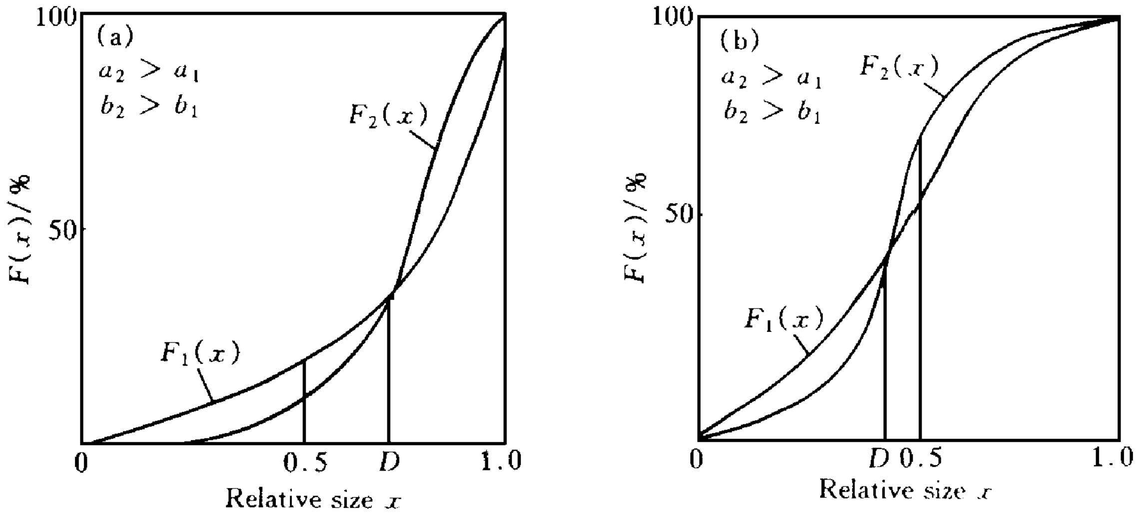

Fig. 3 Schematic illustrations of D-criterion
(a) —$D > 0.5$; (b) —$D < 0.5$

known as a_1, b_1 and a_2, b_2, respectively, we calculate the value of D by use of equation (8) first, and then directly judge the size character of two groups of materials according to D, which is very useful in industrial practice.

3 DETERMINATION OF THE APPRAISABLE FUNCTION OF SIZE CHARACTERISTICS

Using the criterion of size characteristics determined above, we can only judge whether the particle size of a material is coarse or fine. In the theoretical study of mineral processing, especially the size analysis, however, it is even more important to judge the size character of any size range in samples. Therefore, it is necessary to seek out an appraisable function that can accurately appraise the size distribution characteristics of the samples.

3.1 The meaning of the appraisable function

If $J(a, b)$ is a single-valued function of size distribution characteristic parameters a, b, and when $x \in (0, 1)$, $J(a, b)$ possesses the same monotonicity as $F(a, b)$, then the function $J(a, b)$ is defined as "the appraisable function of size characteristics". It is thus clear that the value of $J(a, b)$ can directly show the size distribution of any particle size range in two mineral samples.

3.2 The mathematical model of the appraisable function

Let us introduce a new function $f(a, b)$:

$$f(a, b) = ax^b, x \in (0, 1) \tag{9}$$

Substitute it into equation (2), then:

$$F(a, b) = 1 - \exp(-f(a, b)) \tag{10}$$

It is evident that $f(a, b)$ possesses the same monotonicity as $F(a, b)$.

Equation (9) can be rewritten as:

$$\lg f(a, b) = \lg a + b\lg x \tag{11}$$

Let

$$J(a, b) = \lg f(a, b)$$

Then

$$J(a, b) = \lg a + b\lg x \quad x \in (0, 1) \tag{12}$$

Obviously, $J(a, b)$ in equation (12) is a single-valued function of a, b. When $x \in (0, 1)$, it possesses the same monotonicity as $F(a, b)$ in equation (2), with the correlation shown in Fig. 4, which meets the demands of the appraisable function of size charateristics mentioned in 3.1. Therefore equation (12) is considered as the mathematical model of the appraisable function of size characteristics.

Formula (12) shows that there is different $J(a, b)$ corresponding to different x, so it is also called " generalized appraisable function" or "series of appraisable function", by which we can appraise the size distribution of any particle size range in two specimens.

In accordance with actual conditions and requirements of size analysis, the interval (0, 1) can be divided into several sub-intervals in particular appraisal. Especially, if $x = 0.5$, the mineral particles are divided into two sizes, coarse particle size —— relative size $x \in (0.5, 1)$ and fine particle size —— relative size $x \in (0, 0.5)$. The appraisable function is deduced as:

$$J(a, b) = \lg a - b\lg 2 \quad (13)$$

The values of $J_1(a_1, b_1)$ and $J_2(a_2, b_2)$ can be calculated according to known a_1, b_1 and a_2, b_2. The size distributions of coarse and fine particles in two specimens can be appraised by comparing the value of $J_1(a_1, b_1)$ with that of $J_2(a_2, b_2)$.

When the size distribution of more narrow size range needs to be appraised, it can be efficiently carried out just by substituting the upper limit value of the relative size of the particle size range into x in equation (12).

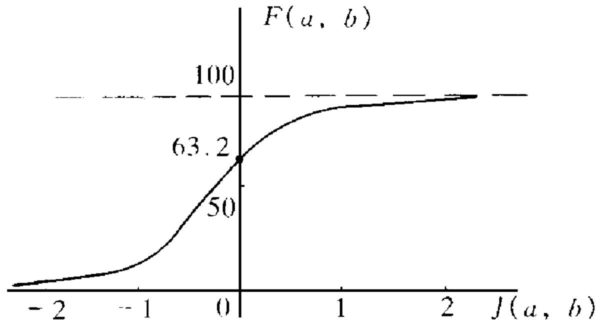

Fig. 4 The correlation between $F(a, b)$ and $J(a, b)$

4 APPLICATION OF THE APPRAISABLE FUNCTION

In practice, when the characteristic parameters a and b vary, for two samples $M_1(a_1, b_1)$ and $M_2(a_2, b_2)$, the values of their appraisable functions are respectively as follows:

$$J_1(a_1, b_1) = \lg a_1 + b_1 \lg x$$

$$J_2(a_2, b_2) = \lg a_2 + b_2 \lg x$$

The difference between them is:

$$\Delta J(a, b) = J_2(a_2, b_2) - J_1(a_1, b_1)$$

$$\Delta J(a, b) = (\lg a_2 - \lg a_1) + (b_2 - b_1)\lg x \quad (14)$$

The following conclusions can be derived from equation (14) as in Table 1.

Table 1 Application of the appraisable function

No.	Conditions	Sign of $\Delta J(a, b)$	Conclusions
1	$a_2 > a_1$, $b_2 = b_1$	$\Delta J(a, b) > 0$	M_1 is coarser than M_2
2	$a_2 > a_1$, $b_2 < b_1$	$\Delta J(a, b) > 0$	
3	$a_2 = a_1$, $b_2 > b_1$	$\Delta J(a, b) < 0$	M_2 is coarser than M_1
4	$a_2 < a_1$, $b_2 > b_1$	$\Delta J(a, b) < 0$	
5	$a_2 > a_1$, $b_2 > b_1$	The sign of $\Delta J(a, b)$ depends on x	Further comparison is needed to different particle size ranges
6	$a_2 < a_1$, $b_2 < b_1$		

From Table 1, it can be found that the conclusions from No. 1 to No. 4 obtained by use of the appraisable function are the same as those before. When both a and b either increase or decrease, the sign of $\Delta J(a, b)$ depends upon the relative size x. The size nature of different particle size range can be judged through further comparison of different x' s.

The data of particle size analysis of two specimens (grinding products) are shown in Table 2 as an example to illustrate the practical application of the appraisable function.

Table 2 The results of size analysis for two samples

Sieve size range/ μm	Relative size x	Cumulative undersize/ %	
		$F_1(x)$	$F_2(x)$
-417+295	1.000	100.00	100.00
-295+208	0.707	97.96	98.77
-208+147	0.500	92.67	89.22
-147+104	0.354	82.74	69.90
-104+74	0.250	69.30	46.70
-74+53	0.177	54.90	28.33
-53+38	0.125	41.18	16.06
-38	0.088	29.85	8.78

On the basis of least square method[7], the optimum solutions of characteristic parameters a and b of size distribution are evaluated from equation (1) as:

$$a_1 = 5.80, \ b_1 = 1.15$$

$$a_2 = 8.20, \ b_2 = 1.85$$

Let $x = 0.5$, from formula (14) we obtain:

$$\Delta J(a, b) = J_2(a_2, b_2) - J_1(a_1, b_1)$$

$$= (\lg 8.20 - \lg 5.80) + (1.85 - 1.15)\lg 0.5 = -0.0603 < 0$$

It can be concluded from above that the fraction of fine size range ($x < 0.5$) in M_1 is more than that in M_2.

On the other hand, we can derive D from formula (8):

$$D = (a_1/a_2)^{[1/(b_2-b_1)]} = (5.80/8.20)^{[1/(1.85-1.15)]} = 0.6098 > 0.5$$

Since $a_2 > a_1$ and $b_2 > b_1$, we can also conclude that the fraction of fine particle in M_1 is more than that in M_2. Thus the conclusion drawn from the criterion of size characteristics is identical with that from the appraisable function of size characteristics and fits with the results of size analysis above.

To appraise grain size distribution of a specific size range, for instance −200 mesh, in two samples, just let $x = 0.177$ and then substitute it into equation (14):

$$\Delta J(a, b) = (\lg 8.20 - \lg 5.80) + (1.85 - 1.15)\lg 0.177 = -0.3760 < 0$$

It can be judged from $\Delta J(a, b) < 0$ that the fraction of −200 mesh in M_1 is greater than that in M_2.

Similarly, the size distribution characteristics of any particle size range in two specimens can be evaluated by use of the appraisable function, which is of important significance in practice and process automation.

5 CONCLUSIONS

(1) When relative size $x \in (0, 1)$, the expression of the criterion of size characteristics is $D = (d_1/d_2)^{[1/(b_2-b_1)]}$. If $D > 0.5$, the bigger the parameters a and b are, the greater the fraction of coarse size is. If $D < 0.5$, the bigger the parameters a and b are, the greater the fraction of fine size is.

(2) The mathematical model of the appraisable function of size characteristics is $J(a, b) = \lg a + b\lg x$. If $\Delta J(a, b) > 0$, for any x, the fraction of the particles whose relative size is less than x in a group of materials with bigger a and b is greater than that with less a and b. And if $\Delta J(a, b) < 0$, the latter is greater than the former.

(3) By using the appraisable function of size characteristics, we can evaluate the size distribution characteristics of any particle size range in two groups of material under all circumstances of a and b.

(4) The criterion and the appraisable function of size characteristics deduced in this paper can be widely used in theory and practice of comminution engineering and mineral processing.

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