# RADIAL EXTRUSION PROCESS BY THREE-DIMENSIONAL RIGID-PLASTIC FEM SIMULATION<sup>®</sup>

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**ABSTRACT** Based on systematic integration, the effective methods to deal with the specific problems were proposed, and the three-dimensional rigid-plastic FEM simulation system, Simu III, was set up. Furthermore, the radial extrusion process was simulated, and the deforming pattern was analyzed. The simulation results fit well with the experimental results, which proves the system's reliability.

Key words radial extrusion three-dimensional rigid-plastic FEM numerical simulation

#### 1 INTRODUCTION

Radial extrusion is a recently-developed extrusion technology which presses the billet along its axial direction to make the material flow in the directions perpendicular to its axial direction, the die with simple structure can be used to finish the forming process which can not be achieved easily by ordinary forming technology, such as the universal joint. Because the part has no flash during forming process, much material can be saved with high manufacturing efficiency at the same time, therefore it has wide applications.

Ref. [1] systematically studied the radial extrusion's classification, the die structure's features, the extrusion deformation, the forming defect and the calculation of extrusion force, and simulated the process of forming the part with side sticks by rigid-plastic FEM. Ref. [2] adapted the rigid-plastic FEM to simulate the triconnecting-tube's radial extrusion, while Refs. [3] and [4] simulated radial extrusion's similar technology—side extrusion. All those works are very useful to analyse the material's flowing pattern and to check the design's reliability, and also to expand the application scope of the numerical simulation technique.

It should be pointed out that the present

work is based on the two-dimensional simulation, and the radial extrusion is different from the side extrusion in that the later could be simplified into 2-D question with little error, but the former is real 3-D forming pattern; if we use 2-D FEM to simulate, larger error may occur. The 3-D rigid-plastic FEM could simulate this kind of radial extrusion precisely, and provide the force's boundary condition for the die's stress analysis.

This paper will employ the threedimensional rigid-plastic FEM to simulate one kind of radial extrusion with two side sticks, furthermore, check the numerical results by experiment.

# 2 RIGID-PLASTIC FEM FORMULATION

The rigid-plastic FEM uses Markov theory of variational calculus to deal with the deforming material so as to acquire the numerical result. By deploying the penalty-function method, we set up the specific equation as follows:

$$\pi = \iiint \hat{\mathbf{\epsilon}} \cdot \hat{\mathbf{\epsilon}} \, \mathrm{d} \, V + \frac{\alpha}{2} \iiint \hat{\mathbf{\epsilon}}_{ij} \cdot \delta_{ij} \cdot \delta_$$

where  $\alpha$  is the penalty factor.

The deforming part is discreted by many finite elements, and the matrix formulation is  $\pi^{(m)}$  corre created, so in No. m element, sponding to the real velocity field is expressed as

$$\pi^{(m)} = \iiint_{V(m)} \nabla \sqrt{\frac{2}{3} \{u\}^{\mathsf{T}} [K] \{u\}} \cdot dV + \frac{\alpha}{2} \iiint_{V(m)} \{i\}^{\mathsf{T}} \cdot \{C\})^{2} dV - \int_{S_{m}(m)} \{iV\}^{\mathsf{T}} \cdot \{F\} dS$$
 (2)

According to the theory of variational calculus, then

$$\delta \pi = \sum \frac{\delta \pi^{(m)}}{\partial u_i} \delta u_i \tag{3}$$

where  $u_i$  is the node velocity.

Because  $\delta u_i$  is random, so the following equation can be acquired

$$\Sigma \frac{\delta \pi^{(m)}}{\partial u_i} = 0 \tag{4}$$

In eq. (2), we evaluate the first derivative of  $\{u\}$ , then

$$\frac{\partial \pi^{(m)}}{\partial \{u\}} = \frac{2}{3} \iiint_{V(m)} \overline{\hat{\epsilon}} \frac{[K]\{u\}}{\hat{\epsilon}} dV + [M]\{u\} - \{f\}$$
 (5)

where

$$[M] = \alpha \iiint_{V(M)} B J^{\mathsf{T}} \{C\} \{C\}^{\mathsf{T}} [B] \, \mathrm{d} V \quad (6)$$

$$\{f\} = \int_{S_{\mathsf{T}}(M)} [NJ^{\mathsf{T}} \{F\} \, \mathrm{d} S \quad (7)$$

The formulation created from eq. (4) is a non-linear equation in which all node velocities need calculating. When we solve the equation by the linear method, the incremental method and Newton Raphson iteration method are adopted, suppose that

$$\{u\}_n = \{u\}_{n-1} + \{\Delta u\}_n$$
 (8)  
we introduce eq. (8) into eq. (4), and implement the Taylor unfolding at  $\{u\}_n$ . If we in

ment the Taylor unfolding at  $\{u\}_{n=1}$ . If we intercept the linear term, we get

$$\frac{\partial \pi^{(m)}}{\partial \{u\}_n} = \frac{\partial \pi^{(m)}}{\partial \{u\}_{n-1}} + \frac{\partial^2 \pi}{\partial^2 \{u\}_{n-1}} \{\Delta u\}_n \quad (9)$$

To integrate eq. (4), eq. (5), eq. (8) and eq. (9), we can get the values of all the nodes' velocities. In addition,  $\{f\}$  in eq. (5) is related to friction term, the corresponding friction force model is

$$f = mkt = -\frac{2}{\pi}mk \operatorname{tg}^{-1}(\frac{\mathbf{W}_{s}}{A})$$
 (10)

So the friction force functional equation is

$$\mathcal{I}_{S_{F}} = -\iint_{S_{F}} \left( \int_{0}^{W_{s}} mk \, \frac{2}{\pi} \operatorname{tg}^{-1} \left( \frac{W_{s}}{A} \right) \operatorname{d} W_{s} \right) \operatorname{d} S$$
(11)

#### 3 CORE DEALING METHODS DURING **SIMULATION**

Compared with 2-D simulation, 3-D simulation is more complex and difficult to deal with, and to solve the key questions correctly is the base to realize the process simulation. The effective methods to treat the key questions are elaborated as follows.

#### 3. 1 Die cavity's geometric information description

This operation needs the support of geometric modeling system. Based on CAD/CAM/CAE software, Unigraphics II, this paper utilizes UnigraphicsII's advanced development—toolkit USER FUNCTION which has various modeling function, adopts CSG and B-rep to describe the three-dimensional die cavity, so we program the code by using ANSI C language & USER FUNCTION. After we finish the die's geometric modeling, the description data are translated into unique B-rep-NURBS (Non-Uniform Rational B-Spline) free form surface, furthermore, we don't need to operate the specific data, and what we need to do is only to operate its identification pointer ID.

#### 3. 2 Implementation of boundary friction condition

To the boundary element, we consider the friction's effect only on its boundary surface with 4 nodes contacting to the die simultaneously, and suppose that the friction acts on the virtual contacting plane to approximate the real complex free form surface.

### Solution of dynamic boundary updating by 3. 3 using tested geometry modeling algorithms of commercial CAD/ CAM software

This paper uses the algorithm in which a

rate intersects with a three-dimensional solid entity to calculate the time increment  $\Delta t$ , and the free boundary node can contact to the die during the current iteration step.

This paper uses the algorithm which judges the relative positions of two three-dimensional entities to check whether the contacted nodes are still on the die's surface, if not, those nodes should be drawn back to the surface along the directions perpendicular to the die surface.

## 3.4 Visualization of simulation result

Another development of Unigraphics II, toolkit GRIP, could be used to display the simulation results such as deforming mesh, stress and strain distribution, velocity field and load-stroke curve. On the window of Unigraphics II, an executable file programmed by GRIP language runs automatically to get great amount of simulation data and create the visual graph for analysis.

## 3. 5 Development of remeshing module

This paper proposes an effective algorithm, and uses Unigraphics II's submodule GFEM, and a code named Remesh III is programmed to regenerate the hexahedron brick elements for simulating the whole deforming process.

Based on the above methods, a general three-dimensional rigid-plastic/rigid-viscoplastic FEM numerical simulation system, Simu III, is developed to trace any three-dimensional bulk forming process, such as forging, extrusion and rolling<sup>[5]</sup>.

#### 4 ANALYSES OF SIMULATION RESULTS

In order to analyze the material's forming pattern and the die's mechanical condition during radial extrusion process, and to check the die design, this work uses Simu III to simulate the radial extrusion process of a part with two side sticks. The material is lead, and the circular billet's dimension is  $d 30 \text{ mm} \times 50 \text{ mm}$ . The lubricant between the material and the lower die is  $20^{\#}$  oil. The friction factor m is 0.15, and the lead's stress vs strain formulation is illustrated as follows<sup>[2]</sup>:

$$\overline{\sigma} = 11.3 + 33.5 \overline{\epsilon}^{0.5}$$
 (12)

The simplified assembly schematic of the separate lower die is shown in Fig. 1. Fig. 2(a) shows the initial mesh discretion of the billet. During meshing, there are two areas in which the mesh density is different from each other, while there is an interim zone. This type of mesh can decrease the nodes' number to increase the calculation efficiency, at the same time assure the computation precision. While along the billet axis, local mesh densification plan is performed so as to consider the nor uniform deformation, e.g., the region where the material will be forced to flow to has the densified mesh. Hence the plan can improve the precision of FEM simulation to approximate the deformed

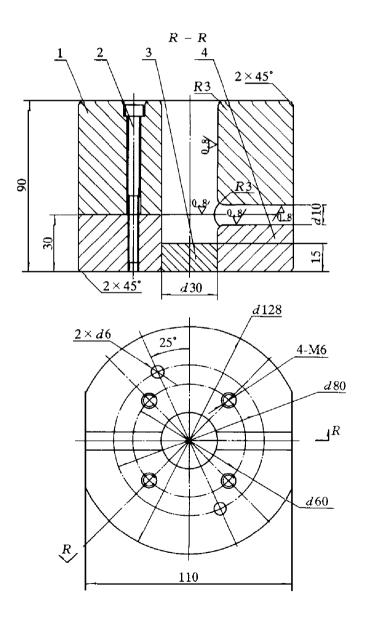


Fig. 1 Simplified assembly schematic of separate lower die

profile.

Figs.  $2(b) \sim (e)$  show the deformed mesh at different strokes, and Fig. (3) shows another views of the deformed mesh.

It is found that radial extrusion is such a technology that the deformation occurs in local region, which is mainly located at the gate of the passage. Therefore, at this region, the mesh is distorted seriously, while the other region's mesh deforms little and it is not easy to find out. Such flow pattern will result in critical mechanical condition in the die cavity, and has the most

serious wear on the die surface, so this region is easy to damage. During the die design, more attentions should be paid to select the tool material or to implement special heat treatment.

# 5 COMPARISON BETWEEN NUMERICAL RESULTS AND EXPERIMENT

In order to verify the simulation code, the corresponding experiment has also been finished on the oil press whose maximum pressure is 490 kN. The die purpose material is 45<sup>#</sup> steel, and

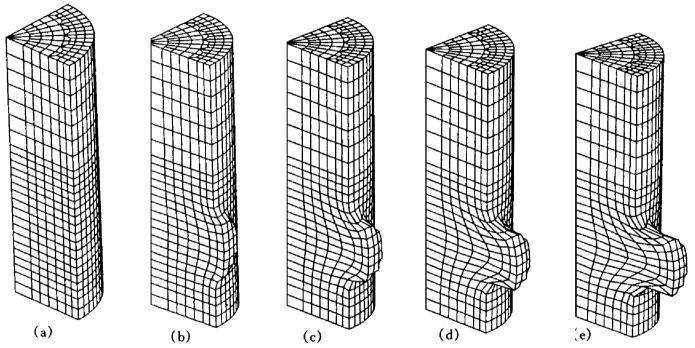


Fig. 2 Billet's initial mesh and deformed mesh at different strokes

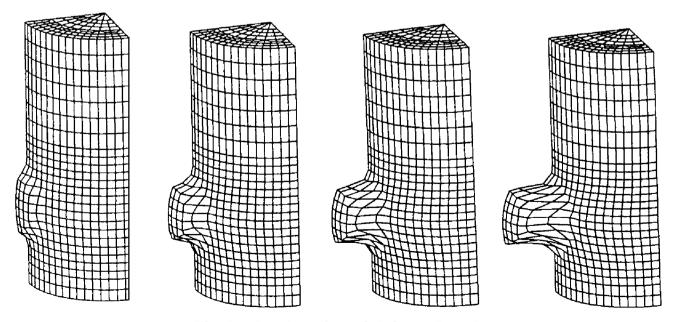


Fig. 3 Another view of deformed mesh

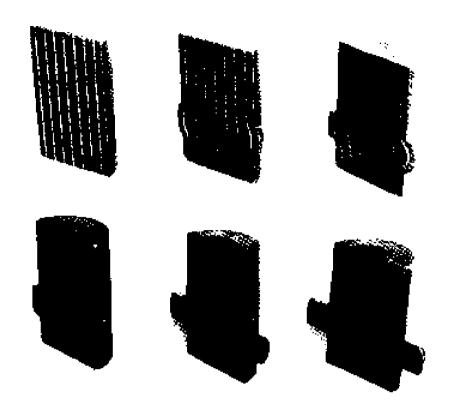


Fig. 4 Experimental results

the lubricant between the billet and the lower die is 20<sup>#</sup> oil. The mesh is engraved on the billet's section, which corresponds to the mesh of simulation for the purpose of comparing the two results easily. Fig. 4 shows the deformed billet at different strokes. It could be found that the simulation of deformed mesh and that of experiment as well as the geometric profile fit well, then the simulation system is proved reliable and can be used to predict the flow behavior instead of the try-out operation.

## 6 CONCLUSION

This paper proposes an effective methods to deal with the main questions in simulating the three dimensional metal forming process, and the simulation code is developed. Then, the new technology—radial extrusion is simulated to ana-

lyze the material's flow pattern. The simulation result and that of experiment coincide well. The simulation software could be used to replace the experiment for predicting the flowing laws and checking the reliability of the design.

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(Edited by Peng Chaoqun)