ELECTROMAGNETIC MODEL OF LEVITATION MELTING WITH COLD CRUCIBLE①

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ABSTRACT The electromagnetic problems of levitation melting with cold crucible were studied. A quasi-three dimensional model of electromagnetic field in this levitation melting process and the modified coupled current method were presented. The influences of crucible structure and power frequency on the electromagnetic field in this process were analyzed.

Key words cold crucible electromagnetic induction levitation melting

1 INTRODUCTION

Levitation melting with cold crucible is a recently developing method to make high melting point, high purity and very active metal or non-metal materials. This method is widely used in metallurgy and special material processing, and a good prospect is shown. This process is that the charge is set in a high frequency magnetic field, and molten by the way of electromagnetic induction; when the gravity of melt is counteracted by the electromagnetic force , it is levitated; hence both the melt and the inner wall of the crucible will not be contacted, and the high temperature and non-pollution will be realized. In this process, cold crucible plays a role of energy concentrator. It can concentrate the source magnetic energy in the limited volume of the crucible to produce high density of magnetic flux for levitating the melt. The key of this technology is the special structure of cold crucible and a reasonable design of electromagnetic field in the melting process. It is demanded that the structure of cold crucible has the characters of low losses of Joule heating and high ability of magnetic penetration, from which the large inducted current and electromagnetic pressure are generated on the melt surface. Hence, the optimization of cold crucible and its theoretical investigation are the important problems to which people pay attention[1-4].

Levitation melting with cold crucible is a comprehensive technology concerning electricity, magnetism, fluid dynamics, heat transfer, mechanics, physicochemistry and metallurgy. Based on the previous work[5], the coupled problem of the structure of crucible and the electromagnetic field in levitation melting process is investigated in this paper. This will establish the fundaments for optimizing design of crucible structure and getting a larger ability of levitation melting with cold crucible.

2 THE STRUCTURE OF COLD CRUCIBLE AND ITS ELECTROMAGNETIC MODEL

A cold crucible is schematically illustrated in Fig. 1. When high frequency current is applied to the inductor, the inducted current will be generated in both the charge and the crucible. This current is a short-circuit current, whose action of heating is similar to that of the secondary coil of a transformer running unloadly. Hence a very large heat source can be generated to melt the charge. Along the axial direction, cold crucible is separated into a number of segments which are isolated each other, so that the mag-

① Supported by the National Natural Science Foundation of China; Received Jun 29, 1995, accepted Dec. 9, 1995

netic field generated by the current in the inductor can penetrate the crucible and work on the charge. In this way, the source magnetic field can not be shielded; and the inducted electropotential generated in the crucible is decreased; also the Joule losses of cold crucible itself in levitation melting is decreased. From these, the levitation melting with cold crucible and the melting with induction furnace are identical in essence, the mere difference of which is that the former includes the electromagnetic induction and heating of crucible itself. Electromagnetically, both ofthem belong to the eddy-current problem.

Fig. 1 The segmented structure of cold crucible

Now if the movement current led by melt flow is neglected, the Maxwell's equations of above problem are

$$\nabla\times\dot{\boldsymbol{E}} = -j\omega\dot{\boldsymbol{B}} \text{ (Farady's law)} \quad (1)$$

$$\nabla\times\dot{\boldsymbol{B}} = \mu\dot{\boldsymbol{J}} \text{ (Ampere's law)} \quad (2)$$

$$\nabla\cdot\dot{\boldsymbol{B}} = 0$$

(magnetic flux continuity law) (3)

and

$$\dot{\boldsymbol{J}} = \sigma\dot{\boldsymbol{E}} \text{ (Ohm's law)} \quad (4)$$

$$\nabla\cdot\dot{\boldsymbol{J}} = 0 \text{ (current continuity law)} \quad (5)$$

where the vectors $\dot{\boldsymbol{E}}$, $\dot{\boldsymbol{B}}$ and $\dot{\boldsymbol{J}}$ are the complex intensity of electric field, complex density of magnetic flux and complex density of current respectively, and ω, μ and σ are the angular frequency of current, permeability and conductivity, and $j = \sqrt{-1}$.

Apply vector potential $\dot{\boldsymbol{A}}$ to above equations, and let $\dot{\boldsymbol{B}} = \nabla\times\dot{\boldsymbol{A}}$, and $\nabla\cdot\dot{\boldsymbol{A}} = 0$, then we can get

$$\dot{\boldsymbol{E}} = -j\omega\dot{\boldsymbol{A}} - \nabla\varphi$$

(φ is the scalar potential) (6)

$$\nabla^2\dot{\boldsymbol{A}} = -\mu\dot{\boldsymbol{J}}$$

(vectorial Poisson equation) (7)

where φ in equation (6) only exists in the outer source zone, while $\dot{\boldsymbol{J}}$ in equation (7) includes all source current and inducted current. The solution of equation (7) can be writen as

$$\dot{\boldsymbol{A}} = \frac{\mu}{4\pi}\int_{\Omega}\dot{\boldsymbol{J}}(\boldsymbol{r})\,d\Omega/|\boldsymbol{r}'| \quad (8)$$

where Ω consists of all the conductive zones. Thus from eqs. (6) and (4), following integral equation can be writen

$$\dot{\boldsymbol{J}} = \frac{j\omega\mu\sigma}{4\pi}\int_{\Omega}\dot{\boldsymbol{J}}(\boldsymbol{r})\,d\Omega/|\boldsymbol{r}'| - \sigma\nabla\varphi \quad (9)$$

To use the direct discreting method and take the current or voltage in source zone as the boundary condition, equation (9) can be solved numerically, then the other variables will be calculated sequently.

$$\dot{\boldsymbol{B}} = (j/(\sigma\omega))\,\nabla\times\dot{\boldsymbol{J}} \quad (10)$$

$$F = \mathrm{Re}(\dot{\boldsymbol{J}}\times\dot{\boldsymbol{B}}^{*})/2$$

(electromagnetic force) (11)

$$P_m = (\dot{\boldsymbol{B}}\cdot\dot{\boldsymbol{B}}^{*})/(4\mu)$$

(electromagnetic pressure) (12)

where the mark * represents conjugate complex. Apparently, the sufficient and necessary condition for the melt to be levitated is that the hydrostatic pressure of the melt is balanced by the electromagnetic pressure generated by alternating magnetic field on the surface of the melt. At the same time, the electromagnetic stirring in the melt will be formed naturally, and its intensity is determined by the rotationary part of electromagnetic force

$$\boldsymbol{F}_s = \frac{1}{2\mu}\mathrm{Re}(\dot{\boldsymbol{B}} \cdot \nabla)\dot{\boldsymbol{B}} \tag{13}$$

3 A MODIFIED COUPLED CURRENT METHOD

The coupled current method is a numerical one to discrete the integral equation (9) directly, and is used in electromagnetic field computation for planar or axisymmetric problems with non-ferromedium such as induction melting and electromagnetic casting[6]. The advantage of this method is that its physical principle is apparent, while its disadvantage is that it can be only used for planar or axisymmetric magnetic field computation in two dimensions. In Fig. 1, one can see that the segmented structure of cold crucible determines that this problem belongs to a type of three dimensional electromagnetic field computation which brings some difficulties to numerical calculation, and results in the coupled current method not to be used directly. Thus, we must simplify the problem and modify the coupled current method to apply to quasi - three dimensional computation.

By calculation and measurement, reference [3] concluded that the current inducted in a crucible can be thought as flowing in horizontal direction: this current flows through the inner and outer surfaces in any vertical cross-section of crucible, and its density in the inner and outer surfaces at the same height of crucible is the same in magnitude, but reversal in direction. From this, the problem can be simplified to the following quasi-three dimensional model: (1) All currents flow horizontally; (2) The three dimensional effect of segmented structure of crucible is considered.

In the quasi-three dimensional model, the character that both source current and inducted current in levitation melting with cold crucible are flowing horizontally determines that the current field in the melt is axisymmetric. While from current continuity law, it is known that the axisymmetric current field is affected by the three dimensional segmented structure of crucible and its inducted current. Hence, for discreted equation (9) with finite elements, inducted current in every melt element can be determined by the current circles of all inductor elements and other melt elements with the same axis, as well as the parallel current circles in all the segments of crucible; and so can be done for the current of each element in inductor and crucible. Thus modified coupled current method can be applied to this quasi- three dimensional electromagnetic field computation. The difference is that the effect of all the parallel current circles in the elements of segmented structure of cold crucible is taken into account.

The process of computation is as follows.

As shown in Fig. 1, a half zone limited by the symmetric axis can be taken as calculating one, in which finite element discretion is carried out. The finite element interpolation relations of the field variables in every element are

$$\dot{\boldsymbol{J}}_e = N_i^{(e)}\dot{\boldsymbol{J}}_i^{(e)},\quad \dot{\boldsymbol{B}}_e = N_i^{(e)}\dot{\boldsymbol{B}}_i^{(e)} \tag{14}$$

Based on the above results, the complex linear equations by discreting equation (9) can be presented as

$$(\boldsymbol{R} + j\omega\boldsymbol{X})\dot{\boldsymbol{I}} = \dot{\boldsymbol{U}} \tag{15}$$

where $\dot{\boldsymbol{I}}$ is the complex vector of node current; $\dot{\boldsymbol{U}}$ is the complex source vector whose element is non-zero for the source zone only. If the voltage U_c between two turns of inductor is taken as the boundary condition, the element of $\dot{U}_i$ can be writen

$$\dot{\boldsymbol{U}}_i = \begin{cases} (U_c^{(m)},\ 0) & \text{(node } i \text{ is in source zone)} \\ (0,\ 0) & \text{(node } i \text{ isn't in source zone)} \end{cases} \tag{16}$$

where m is the number of turns of the inductor. In equation (15), $\boldsymbol{R}$ is the resistance matrix with diagonal form, in which R_{ii} is the resistance of the i-th node current circle; and $\boldsymbol{X}$ is a full and symmetric inductance matrix, whose element in the case of coupled current method is

$$X_{ij} = \begin{cases} L_i & (\text{for } i = j) \\ M_{ij} & (\text{for } i \neq j) \end{cases} \tag{17}$$

where L_i and M_{ij} are the self-inductance coeffecient of i-th node current circle and the mutual inductance coeffecient between i-th and j-th node current circles, separately. While in the quasi-three dimensional computation of electromagnetic field in cold crucible, the effect of the segmented structure of crucible as well as its inducted current must be taken into account. This effect resulted from three dimensional structure of crucible appeared in the matrix $\boldsymbol{X}$ only. Then, for modified coupled current method, the element of matrix $\boldsymbol{X}$ is changed into:

$$X_{ij} = \begin{cases} L_i & (i = j, \text{ and node } i \text{ isn't in crucible}) \\ L_i + \sum_{k=1}^{n-1} M_{ij}^{(k)} & (i = j, \text{ and node } i \text{ is in crucible}) \\ M_{ij} & (i \neq j, \text{ and node } j \text{ isn't in crucible}) \\ \sum_{k=1}^{n} M_{ij}^{(k)} & (i \neq j, \text{ and node } j \text{ is in crucible}) \end{cases} \quad (18)$$

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where n is the number of segments of cold crucible.

From this, the numerical calculation of electromagnetic field in levitation melting with cold crucible can be carried out. In order to decrease the storage in calcaulation, both the vectors $\boldsymbol{I}$ and $\boldsymbol{U}$ can be divided into two parts $\dot{\boldsymbol{I}} = (\boldsymbol{I}_{\mathrm{R}}, \boldsymbol{I}_{\mathrm{m}})^{\mathrm{T}}$ and $\dot{\boldsymbol{U}} = (\boldsymbol{U}_{\mathrm{R}}, \boldsymbol{U}_{\mathrm{m}})^{\mathrm{T}}$, then equation (15) can be extended into two separated real linear equations

$$\left.\begin{aligned} (\boldsymbol{R} + \omega^2 \boldsymbol{X}\boldsymbol{R}^{-1}\boldsymbol{X})\boldsymbol{I}_{\mathrm{R}} &= \boldsymbol{U}_{\mathrm{R}} + \omega \boldsymbol{X}\boldsymbol{R}^{-1}\boldsymbol{U}_{\mathrm{m}} \\ \boldsymbol{R}\boldsymbol{I}_{\mathrm{m}} &= \boldsymbol{U}_{\mathrm{m}} - \omega \boldsymbol{X}\boldsymbol{I}_{\mathrm{R}} \end{aligned}\right\} \quad (19)$$

Thus they can be solved sequently.

4 RESULTS AND ANALYSIS

The numerical analysis of electromagnetic field is carried out for a cold crucible whose inner diameter is 6 cm, volume is 140 cm^3 and levitation ability is about 1 kg. The results of electromagnetic field and electromagnetic force field are illustrated in Fig. 2.

In Fig. 2, it can be seen that during levitation melting with cold crucible, magnetic flux density B in the surface of melt is along the generatrix of melt, and the inducted current J of melt is along the azimuthal direction which is perpendicular to the paper face, hence the electromagnetic force $\boldsymbol{F} = \boldsymbol{J} \times \boldsymbol{B}$ is in the direction of inner normal vector of melt surface. Thus the melt is pushed by the electromagnetic force apart from the inner wall of cold crucible and non-contacted with that. When the hydrostatic pressure of melt is balanced by the electromagnetic force, the melt is levitated. The calculated result also shows that when the gap between the melt surface and the inner wall of crucible is decreased, the intensity of electromagnetic field within the gap can reach a very high level. As a result, both the inducted current and the electromagnetic pressure on the surface of melt are increased, and the melt can be pushed inward. This means that the levitation melting with cold crucible is a self-stable and self-balanced process.

During levitation melting with cold crucible, the frequency of the alternating magnetic field and the structure of cold crucible are the key factors which determine the magnetic penetration of melt, the Joule losses in crucible, the magnetic flux density in melting process and the levitation ability of crucible. In this paper, the effects of the frequency, the number of segments of crucible and the gap between two segments on the magnetic flux density in the crucible volume are calculated and analyzed.

First, the effect of the frequency is presented. Fig. 3 shows the relation between the magnetic flux density at point A in the crucible ($r/R = 0.65$, $Z/H = 0.45$) and the magnetic field frequency. From this figure, it is shown that as the frequency becomes high, the magnetic flux density in the crucible is decreased, and this tendency becomes more prominent when the frequency is over 100 kHz. This indicates that low frequency magnetic field is easy to penetrate the crucible.

Second, the effect of segmented structure of cold crucible is presented. Also in Fig. 3, one can notice that the less the number of the segments is, the more quickly the magnetic flux density decreases as the frequency increases from 10 to 250 kHz. For the crucibles with different numbers of segments ($n = 4 \sim 24$), it is shown that as the magnetic field frequency increases from 10 to 250 kHz, the decreasing magnitude of

Fig. 2 EM field (a) and EM force (b) in levitation melting with cold crucible

Fig. 3 Influence of frequency on magnetic flux density

the magnetic flux density at point A is from 28% (for $n = 4$) to 5% (for $n = 24$), but when the frequency is less than 10 kHz, the magnetic flux density doesn't vary with the number of segments of cold crucible for $n \geqslant 4$. This indicates that the effect of decreasing the Joule losses of crucible itself by increasing the number of segments of crucible is apparent at high frequency, but at low frequency the crucible can be divided into 2~ 4 segments reasonably and increasing the number of segments is useless.

Fig. 4 shows that in the case of the frequency $f = 250$ kHz and the gap between two segments $\delta_s = 1.2$ mm, based on the reference of the magnetic flux density at point A for $n = 4$, the magnification of magnetic flux density at this point increases as the number of segments of crucible increases. It is shown that when the number of segments of crucible increases from 4 to 24, the magnetic flux density at point A increases quickly initially and then gradually becomes slow, and it almost dosen't increase when $n >$ 24. Hence, it is reasonable that the number of segments of crucible be chosen within 16 to 20 when the frequency is high.

Finally, the result of calculation shows that for all the frequencies and the numbers of segments selected, the gap between two segments affects the magnetic flux density in crucible volume less than the above metioned factors. It is shown that the increase of the magnetic flux density at point A is only about 2% for the gaps between two segments from 0.1 mm to 2.2 mm.

Fig. 4 Influence of segmented structure of cold crucible on magnetic flux density

Thus, it is important to control the gap between two segments for insulating, but when the insulating condition is satisfied, the effect of increasing the gap is not apparent.

5 CONCLUSIONS

(1) Levitation melting with cold crucible is a recently developing method to make high melting point, high purity and very active metal or non-metal materials. Magnetohydrodynamically, this process is self-balanced and self-stable.

(2) During levitation melting with cold crucible, the effect of magnetic field frequency on the magnetic flux density is very large, which indicates that the magnetic flux density in crucible decreases as the frequency increases.

(3) The segmented structure of cold crucible makes the Joule losses generated in the crucible itself become low, and the higher the magnetic field frequency is, the more apparent the effect of the segmented structure is. However, for low frequency, the key problem is the segmented structure of cold crucible, but the effect of the number of segments selected in this paper is not apparent.

(4) In the segmented structure of cold crucible, the gap between two segments must satisfy the insulation conditions of there, and the effect of increasing the gap on the increase of magnetic flux density in the cold crucible is not apparent.

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(Edited by Peng Chaoqun)