CURVILINEAR INTEGRAL OF THE VELOCITY FIELD OF DRAWING AND EXTRUSION THROUGH ELLIPTIC DIE PROFILE①

Zhao, Dewen   Zhao, Hongjin   Wang, Guodong
Rolling Technology and Tandem Rolling Automation National Key Lab, Northeastern University, Shenyang 110006
Fang, Youkang
Institut für Angewandte Mathematik, Albert-Ludwigs-Universitat, BRD

ABSTRACT The kinematically admissiable velocity and strain rate fields were established to extrusion and drawing through the elliptic die profile. The curvilinear integral and the integral as a function of the upper limit were used respectively in obtaining the contact friction and plastic deformation powers. Furthermore, an upper bound analytical solution of deforming force was got for the plane strain drawing and extrusion through the elliptic die.

Key words die with elliptic profile   curvilinear integral   analytical solution

1 INTRODUCTION

To the problems of drawing through a wedge shaped die(the contour line of the die is a straight), corresponding upper bound solutions were discussed in Refs. [1—3].

However, when the contour line of the profile is not a straight one but a complicated mathematic curve, getting an analytical solution will be even more difficult. The approximate solutions with the slip line and the triangular velocity field were reported in Refs. [4—5].

The purpose of this paper is to establish continuous velocity field, as the same method in Refs. [1—3], for the drawing and extrusion through the curved dies.

Then emphatically, the curvilinear integral and integral as a function of the upper limit will be used in seeking for possibility of analytical solution. It seems that about plane strain drawing through the elliptic die, no more analytical solutions in continuous velocity field have been reported.

2 EQUATION OF THE DIE PROFILE

The deforming zone of plane strain drawing and extrusion through an elliptic die is shown in Fig. 1. With Von Karman's basic assumptions, that is, the direction of the applied load and planes perpendicular to this direction define the principal directions, and the

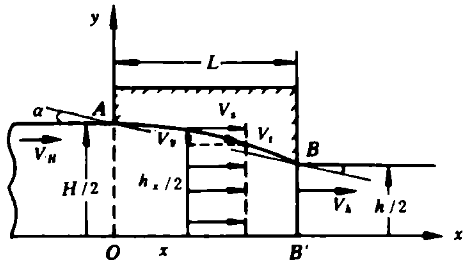

Fig. 1 Plane strain drawing through an elliptic die

① Received Oct. 20, 1994; accepted Dec. 14, 1994

drawing stresses do not vary on these planes. In Fig. 1, the deformation is in plane strain, with no change in the width. Only upper-half-plane of deforming zone above horizontal symmetric axis is illustrated.

For drawing, the prescribed velocity at exit of deforming zone is v_h, corresponding thickness is $h = 2BB'$; let velocity at entry be v_H, the thickness be $H = 2AO$, and the origin of the coordinates be at the entry on the horizontal axis. Then, the equation of elliptic profile of the die for $Y = h_x/2$ is:

$$L^2y^2 + [(\frac{H}{2})^2 - (\frac{h}{2})^2]x^2 = L^2(\frac{H}{2})^2$$

$$\frac{h}{2} \leqslant y \leqslant \frac{H}{2}, 0 \leqslant x \leqslant L$$

or: $$L^2h_x^2 + (H^2 - h^2)x^2 = L^2H^2 \quad (1)$$

From formula (1), the thickness of cross section at the distance x from entry is:

$$h_x = \frac{\sqrt{L^2H^2 - (H^2 - h^2)x^2}}{L} \quad (2)$$

In formula (2), when $x = 0$, $h_x = H$; $x = L$, $h_x = h$. The h_x satisfies the condition at entry and exit of deforming zone, and L is the projection of the contact arc on the horizontal axis.

3 VELOCITY AND STRAIN RATE FIELDS

From Fig. 1, for unit width in deforming zone the volume constancy satisfies the following equation:

$$v_h \cdot h = v_H \cdot H = v_x \cdot h_x = C \quad (3)$$

Where C is the fixed rate flow per second in unit width. For drawing, $C = v_h h$; for extrusion, $C = v_H H$; in the deforming zone, C is independent of x. the v_x is horizontal velocity on cross section h_x at the distance x from entry. From Eqs. (2) and (3), we can get:

$$v_x = \frac{C}{h_x} = \frac{LC}{\sqrt{L^2H^2 - (H^2 - h^2)x^2}} \quad (4)$$

From Cauchy equation, or geometric Eqs., $\varepsilon_x = \frac{\partial v_x}{\partial x}$, we can get:

$$\dot{\varepsilon}_x = -\dot{\varepsilon}_y = \frac{\partial v_x}{\partial x}$$

$$= \frac{LC(H^2 - h^2)x}{\sqrt{[L^2H^2 - (H^2 - h^2)x^2]^3}} \quad (5)$$

Noticing Karman's assumptions, when $i \neq j$, $\dot{\varepsilon}_{ij} = 0$. From Canchy Eqs., we have:

$$v = \int \dot{\varepsilon}_y \mathrm{d}y = \int -\frac{LC(H^2 - h^2)x}{\sqrt{[L^2H^2 - (H^2 - h^2)x^2]^3}}\mathrm{d}y$$

$$\left.\begin{aligned} v_y &= -\frac{LC(H^2 - h^2)xy}{\sqrt{[L^2H^2 - (H^2 - h^2)x^2]^3}} \\ v_x &= \frac{LC}{\sqrt{L^2H^2 - (H^2 - h^2)x^2}} \end{aligned}\right\} \quad (6)$$

Substituting $x = 0$ into the second formula in Eq. (6) yields $v_x = C/H = v_H$; and $x = L$ into it, $v_x = C/h = v_h$. Substitute $y = 0$ into the first formula in Eq. (6) yields $v_y = 0$. So Eq. (6) satisfies the velocity boundary conditions at the exit, entry and horizontal symmetric axis. Notice that in formula (5) $\dot{\varepsilon}_x + \dot{\varepsilon}_x = 0$, Eqs. (5) and (6) are kinematically admissible velocity and strain rate fields.

4 DEDUCTION ON THE FORMULA OF DEFORMING FORCE

Referring to Ref. [3], the internal power of deformation for unit width is:

$$\dot{W}_i = 2k\int_v \sqrt{\frac{1}{2}\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}}\mathrm{d}v = 2k\int_v \dot{\varepsilon}_x \mathrm{d}x\mathrm{d}y$$

$$= 4k\int_0^{\frac{h_x}{2}}\int_0^L \frac{LC(H^2 - h^2)x}{\sqrt{[L^2H^2 - (H^2 - h^2)x^2]^3}}\mathrm{d}x\mathrm{d}y$$

Notice $y = h_x/2$, and upper limit of above integral is still a function of x, so it is just an integral as a function of the upper limit. Substituting (2) into it yields:

$$\begin{aligned} \dot{W}_i &= 4k\int_0^L \frac{LC(H^2 - h^2)x\mathrm{d}x}{\sqrt{[L^2H^2 - (H^2 - h^2)x^2]^3}}\int_0^{\frac{h_x}{2}}\mathrm{d}y \\ &= 4k\int_0^L \frac{C(H^2 - h^2)x\mathrm{d}x}{2[L^2H^2 - (H^2 - h^2)x^2]} \\ &= Ck\int_0^L -\frac{\mathrm{d}[L^2H^2 - (H^2 - h^2)x^2]}{[L^2H^2 - (H^2 - h^2)x^2]} \\ &= -Ck\ln[L^2H^2 - (H^2 - h^2)x^2]_0^L \\ &= Ck\ln\frac{L^2h^2}{L^2H^2} = -Ck\ln(\frac{h}{H})^2 \\ &= 2Ck\ln\frac{H}{h} \end{aligned} \quad (7)$$

In metal forming, $\mu = H/h$ is called coefficient of elongation; k is the yielding shear stress of deformed metal. Formula (7) is the internal plastic deformation power of unit width for drawing through the elliptic die.

Substituting $x = 0$ into the first formula in(6), then, at the section OA, $v_y = 0$, so along the section OA at entry no shear power will be consumed. Substituting $x = L$ into the first formula in(6), along cross section BB' at exit we have:

$$v_y = -\frac{L^2C(H^2 - h^2)y}{\sqrt{[L^2H^2 - L^2H^2 + L^2h^2)]^3}} = -\frac{C(H^2 - h^2)y}{Lh^3} \tag{8}$$

It shows that the shear power must be consumed along section BB' at the exit. Notice that, in Fig. 1, right side of the section BB' is rigid region, so along its tangential direction:

$$|\Delta v_t| = |0 - v_y| = \frac{C(H^2 - h^2)}{Lh^3}y$$

Thus, the shear power along cross section at exit for unit width is:

$$\dot{W}_S = \int_S |\Delta v_t| \cdot k \cdot \mathrm{d}S = 2k\int_0^{BB'}\frac{C(H^2 - h^2)}{Lh^3}y\mathrm{d}y = 2k\int_0^{h/2}\frac{C(H^2 - h^2)}{Lh^3}y\mathrm{d}y = 2kC\frac{C(H^2 - h^2)}{Lh^3}(\frac{1}{2}y^2)_0^{h/2} = \frac{kC(H^2 - h^2)}{4Lh} \tag{9}$$

From Fig. 1, we know that the elliptic arc $\overset{\frown}{AB}$ is the contact interface. Let the friction stress on it be $\tau_f = mk$; the tangential velocity of deformed metal along the arc $\overset{\frown}{AB}$ is v_t. Since the die is stationary, along the tangential direction of interface $\overset{\frown}{AB}$:

$$|\Delta v_t| = |O - v_t| = |v_t|$$

Therefore, the total friction powers consumed along the upper and lower interfaces for unit width is:

$$\dot{W}_f = 2\int_S \tau_f|\Delta v_t|\mathrm{d}S = 2\int_0^{\overset{\frown}{AB}} mk|v_t|\mathrm{d}S \tag{10}$$

Noticing both friction stress τ_f and velocity of metal v_t are on the tangential direction of the interface, the above integral is the curvilinear integral along the elliptic arc $\overset{\frown}{AB}$. The modulo of the tangential velocity vector $\boldsymbol{v}_t$ at the interface is $|\boldsymbol{v}_t| = \sqrt{v_x^2+v_y^2}$. Taking the following equation

$$y = \frac{h_x}{2} = \frac{\sqrt{L^2H^2 - (H^2 - h^2)x^2}}{2L}$$

into the first formula in Eq. (6), we can get:

$$v_y|_{y=\frac{h_x}{2}} = -\frac{C(H^2 - h^2)x}{2[L^2H^2 - (H^2 - h^2)x^2]} \tag{a}$$

Substitute (a) and the second formula in (6) into the following formula, then rearrange it:

$$|\boldsymbol{v}_t| = \sqrt{v_x^2+v_y^2} = [(\frac{LC}{\sqrt{L^2H^2-(H^2-h^2)x^2}})^2 + (\frac{-C(H^2-h^2)x}{2[L^2H^2-(H^2-h^2)x^2]})^2]^{\frac{1}{2}} = \frac{LC}{\sqrt{L^2H^2-(H^2-h^2)x^2}} \cdot \{1+[\frac{(H^2-h^2)x}{2L\sqrt{L^2H^2-(H^2-h^2)x^2}}]^2\}^{\frac{1}{2}} \tag{b}$$

From(2), since $y = \frac{h_x}{2}$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{(H^2 - h^2)x}{2L\sqrt{L^2H^2 - (H^2 - h^2)x^2}} \tag{c}$$

From Eq. (c), we have:

$$\mathrm{d}S = \sqrt{1 + (\frac{\mathrm{d}y}{\mathrm{d}x})^2}\mathrm{d}x = \sqrt{1 + [\frac{(H^2 - h^2)x}{2L\sqrt{L^2H^2 - (H^2 - h^2)x^2}}]^2}\mathrm{d}x \tag{d}$$

Substitute(b) and(d) into(10), and notice $S = 0$, $x = 0$; $s = \overset{\frown}{AB}$, $x = L$, then:

$$\dot{W}_f = 2mk\int_0^L \frac{LC}{\sqrt{L^2H^2 - (H^2 - h^2)x^2}} \cdot \{1 + [\frac{(H^2 - h^2)x}{2L\sqrt{L^2H^2 - (H^2 - h^2)x^2}}]^2\}\mathrm{d}x = 2mkLC\int_0^L \frac{\mathrm{d}x}{\sqrt{L^2H^2 - (H^2 - h^2)x^2}} + 2mkC\int_0^L \frac{(H^2 - h^2)x^2\mathrm{d}x}{4L\sqrt{[L^2H^2 - (H^2 - h^2)x^2]^3}} \tag{e}$$

From Ref. [6],

$$\int \frac{1}{\sqrt{ax^2 + c}}\mathrm{d}x = \frac{1}{\sqrt{-a}}\arcsin(x\sqrt{\frac{-a}{c}})$$

$$(a < 0)$$

Hence, the result of the first integral term in(e) is given by:

$$2mkLC\int_0^L \frac{dx}{\sqrt{L^2H^2-(H^2-h^2)x^2}}$$
$$= \frac{2mkLC}{\sqrt{H^2-h^2}}\arcsin\left(x\sqrt{\frac{H^2-h^2}{L^2H^2}}\right)\Big|_0^L$$
$$= \frac{2mkLC}{\sqrt{H^2-h^2}}\arcsin\left(\sqrt{\frac{H^2-h^2}{H^2}}\right) \qquad (f)$$

From Ref. [7],

$$\int \frac{x^2}{\sqrt{(ax^2+c)^3}}dx = -\frac{x}{a\sqrt{ax^2+c}} + \frac{1}{a\sqrt{-a}}\arcsin\left(x\sqrt{\frac{-a}{c}}\right) \quad (a<0)$$

The result of the second integral term in formula(e) is given by:

$$\frac{2mkC(H^2-h^2)^2}{4L} \cdot \int_0^L \frac{x^2}{\sqrt{[L^2H^2-(H^2-h^2)x^2]^3}}dx$$
$$= \frac{mkC(H^2-h^2)^2}{2L} \cdot \left[\frac{x}{(H^2-h^2)\sqrt{L^2H^2-(H^2-h^2)x^2}} + \frac{1}{-(H^2-h^2)\sqrt{H^2-h^2}} \cdot \arcsin\left(x\sqrt{\frac{H^2-h^2}{L^2H^2}}\right)\right]_0^L$$
$$= \frac{mkC(H^2-h^2)^2}{2L}\left[\frac{1}{h(H^2-h^2)} - \frac{1}{(H^2-h^2)\sqrt{H^2-h^2}} \cdot \arcsin\left(\sqrt{\frac{H^2-h^2}{H^2}}\right)\right]$$
$$= \frac{mkC(H^2-h^2)^2}{2L}\left[\frac{1}{h} - \frac{1}{\sqrt{H^2-h^2}} \cdot \arcsin\left(\sqrt{\frac{H^2-h^2}{H^2}}\right)\right] \qquad (g)$$

Substituting the results of integration(f) and (g) into the formula(e), then rearrange it:

$$\dot{W}_f = \frac{2mkLC}{\sqrt{H^2-h^2}}\arcsin\left(\sqrt{\frac{H^2-h^2}{H^2}}\right) + \frac{mkC(H^2-h^2)}{2L}\left[\frac{1}{h} - \frac{1}{\sqrt{H^2-h^2}} \cdot \arcsin\left(\sqrt{\frac{H^2-h^2}{H^2}}\right)\right]$$
$$= \frac{mkC(H^2-h^2)}{2Lh} + \frac{4mkL^2C - mkC(H^2-h^2)}{2L\sqrt{H^2-h^2}} \cdot \arcsin\sqrt{\frac{H^2-h^2}{H^2}} \qquad (10)'$$

The formulas (10)′ and (10) are the curvilinear integral results.

For drawing, with the upper bound theorem, let applied external power at the exit per unit width be $\dot{W}_d = \sigma_f \cdot v_h \cdot h = \sigma_f \cdot C = \dot{W}_i + \dot{W}_s + \dot{W}_f$; substituting(7), (9) and (10′) into it and rearranging, the analytical solution to the ratio of drawing stress to $2k$ for elliptic die per unit width is given by:

$$\frac{\sigma_f}{2k} = \ln\frac{H}{h} + \frac{H^2-h^2}{8Lh} + \frac{m(H^2-h^2)}{4Lh} + \frac{4mL^2-m(H^2-h^2)}{4L\sqrt{H^2-h^2}} \cdot \sin^{-1}\left(\sqrt{\frac{H^2-h^2}{H^2}}\right) \qquad (11)$$

For extrusion, let the exerted power at entry be given by $W_e = \sigma_e \cdot v_H \cdot H = \sigma_e \cdot C = \dot{W}_i + \dot{W}_s + \dot{W}_f$; substitute(7), (9) and (10′) into the formula and rearrange it, then analytical solution of the ratio of plane strain extrusion stress to $2k$ for elliptic die per unit width is:

$$\frac{\sigma_e}{2k} = \ln\frac{H}{h} + \frac{H^2-h^2}{8Lh} + \frac{m(H^2-h^2)}{4Lh} + \frac{4mL^2-m(H^2-h^2)}{4L\sqrt{H^2-h^2}} \cdot \sin^{-1}\left(\sqrt{\frac{H^2-h^2}{H^2}}\right) \qquad (12)$$

In the above formulas, H, h and L are given parameters of the die(see Fig. 1); m is an experimental constant called constant friction factor. It can be measured or calculated by suggested following formula[8]:

$$m = f\left[1 + \frac{1}{4} \cdot \frac{L}{\bar{h}}(1-f)^4\sqrt{f}\right] \qquad (13)$$

where f is the slip friction coefficient; $\bar{h} = (H+h)/2$.

Joining A and B in Fig. 1 yields the deforming zone $ABB'O$ for plane-strain wedge drawing. It can be seen from Fig. 1 that the geometrical relationship between wedge-shaped and elliptic dies is given by:

$$L = \Delta h/2\tan\alpha \qquad (H)$$

where α is the semi-die angle for the corresponding wedge drawing.

Example: A sheet of metal with initial thickness of 2.54 mm is to be drawn to 2.286 mm respectively through a wedge shaped and an elliptic die with included angle 30°. If an average value for the coefficient of friction is 0.08, calculate the value of $\sigma_f/2k$ needed to complete the operation.

Solution: Using Eq. (13) and noting that $\bar{h} = 2.413$ mm, $\Delta h = 0.254$ mm, $f = 0.08$, $L = 0.254/2\tan 15° = 0.474$ mm, then we get the value $m = 0.082$. Substituting $m = 0.082$, $L = 0.474$, $H = 2.54$ mm, $h = 2.286$ mm into Eq. (11), the value of $\sigma_f/2k$ for the elliptic die drawing yields:

$$\sigma_f/2k = 0.264$$

However, for plane strain wedge drawing using Eq. (8-33) from Ref. [9], the upper-bound value of $\sigma_f/2k$ to the same reduction is:

$$\sigma_f/2k = (1 + m/\sin 2a)\varepsilon + \tan\alpha/2$$

Substituting $\varepsilon = \ln(H/h) = 0.1054$, $m = 0.082$, $a = 15°$ into above formula yields:

$$\sigma_f/2k = 0.257$$

It can be seen that the relative error between them is only:

$$\Delta = (0.264 - 0.257)/0.264 = 2.6\%$$

5 THE LIMITING DRAWING REDUCTION

For metal plastic processing, when $\sigma_f = 2k$, the maximum possible drawing stress is reached. Based on the condition, the corresponding maximum drawing reduction from (11) is given by:

$$\ln\frac{H}{h} + \frac{H^2 - h^2}{8Lh} + \frac{m(H^2 - h^2)}{4Lh} + \frac{4mL^2 - m(H^2 - h^2)}{4L\sqrt{H^2 - h^2}} \cdot \sin^{-1}\left(\sqrt{\frac{H^2 - h^2}{H^2}}\right) \leqslant 1 \qquad (14)$$

6 CONCLUSIONS

(1) With Karman's basic assumptions, the kinematically admissible velocity and strain rate fields in rectangular coordinates for extrusion and drawing through elliptical die satisfy the formulas (5) and (6) in this paper.

(2) To the above fields, an upper bound analytical solution of drawing and extrusion stress is obtained by the curvilinear integral and the integral as a function of the upper limit. The solution is the function of yielding shear stress k, friction factor m, die parameters H, h and L.

(3) The limiting drawing reduction satisfies the formula (14).

(4) Formulas (11), (12) and (14) can be references for calculating deforming forces in extrusion and drawing through the elliptic die profile.

(5) The integrations in the paper can be for references in researching deforming force through complicated die profile.

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