

Critical blank-holder force in axisymmetric deep drawing

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Abstract: An analytical approach for predicting the critical blank holding force (BHF) was presented. Using energy method, the analysis provides the circumferential stress and the equivalent strain as functions of radius under the plane strain and the equivalent strain is inversely proportional to the radius respectively. The maximum relative errors of the circumferential stress and the equivalent strain are 22.3% and 35.9% respectively under the two conditions for some dimensions of sheet and die. In addition, the relationship between BHF and wrinkle number was obtained under the assumption that wrinkle shape is expressed by power function. The critical BHF under plane strain was analyzed for the wrinkle shapes when the power is less than, equal to or greater than 1. The effects of wrinkle shapes on the critical BHF are also presented.

Key words: sheet metal forming; deep drawing; wrinkling; critical blank-holder force; wrinkle model; wrinkle number

1 Introduction

Wrinkling is an instability phenomenon that may occur in sheet metal forming. The analysis of wrinkling is the basis of the control of BHF.

Wrinkling instability is usually analyzed using bifurcation theory. HILL's bifurcation and uniqueness theory initiated the general analytical study of plastic wrinkling [1]. HUTCHINSON [2] developed the bifurcation theory for structures in the plastic range. Based on this work, HUTCHINSON and NEALE [3] later studied the buckling behavior of doubly-curved sheet metal. However, BHF is not considered in the research. TRIANTAFYLIDIS and NEEDLEMAN [4] studied this problem and analyzed the effect of blank holder stiffness on the critical buckling stress and wave number. The energy method is another approach. SENIOR [5] presented a one dimensional model and gave a wrinkling criterion using energy method, considering sheet thickness to be constant. Assumptions are made to determine the curve of the critical BHF vs punch stroke. YU and JOHNSON [6] used a two-dimensional buckling model of an elastic-plastic

annular plate to determine the critical conditions and also quantitatively investigated the effects of a blank holder on the critical buckling stress and wave number. LIN et al [7] derived models of critical fracture and critical wrinkling respectively and discussed the formability of aluminium alloy sheet in the case of variable BHF. GAO et al [8] gave a formula of the critical BHF required to avoid wrinkling in the flange region accounting for friction. ZHAO [9] discussed the effect of friction coefficient and anisotropy coefficients on the critical BHF. WANG [10] investigated BHF through the theoretical analysis and experiment. Based on the work of WANG [10], LUO [11] conducted the analysis to obtain the safe region of BHF which is between the critical rupturing curve and the critical wrinkling curve. YAGAMI et al [12] investigated the effect of blank holder motion on deep drawability and showed that wrinkle elimination can be successful if wrinkles are within the allowable height range. WANG and CAO [13] studied the critical wrinkling condition of a square cup forming under the constant BHF assumption. AGRAWAL et al [14] derived a formula of the critical BHF required to avoid wrinkling of the flange in axisymmetric deep drawing process, considering the

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influence of thickness. SIVASANKARAN et al [15] used an artificial neural network model for predicting and avoiding surface failures such as wrinkling of sheet metals.

To simplify the calculation of strain energy, many researchers have made the assumption that equivalent strain is inversely proportional to radial coordinate by using energy method. The strain energy refers to the energy due to bending deformation and circumferential stress when wrinkling happens. However, such calculated critical BHF is not in consistence with experiment. In order to calculate the BHF accurately, the assumption of plane strain is utilized to analyze the stress and strain. The critical condition for wrinkling is given and the critical BHF is calculated. Since wrinkle shape has a great influence on the critical BHF, the curves of critical BHF vs punch stroke are calculated for different assumed wrinkling models.

2 Analysis of strain and stress in flange

2.1 Plane strain assumption

According to the characteristics of the axisymmetric deep drawing, the following assumptions are made.

1) Plane deformation in the flange, i.e., the strain, is zero in the thickness direction. It can be expressed as $\varepsilon_z=0$.

2) The equivalent stress and equivalent strain satisfy the relationship $\sigma=B\varepsilon^n$, where B and n are strength coefficient and hardening exponent, respectively.

3) The frictions between blank holder and sheet, and between die and sheet are neglected.

4) Bending and reverse bending are not considered. namely, the sheet bending deformation on the corner of the punch and the die is neglected.

Plane strain assumption has been used to analyze the flange deformation in axisymmetric deep drawing. Differential equilibrium equation can be expressed as

$$d\sigma_\rho = (\sigma_\theta - \sigma_\rho) \frac{d\rho}{\rho} \quad (1)$$

where σ_ρ and σ_θ denote the normal stress in the radial direction and the normal stress in the circumferential direction respectively at $r=\rho$; $d\sigma_\rho$ represents the radial stress increment when the radius increases $d\rho$ at the time t .

According to the plane strain assumption and the definition of equivalent stress, the relationship between σ_ρ and σ_θ can be obtained as

$$\sigma_\rho - \sigma_\theta = \sqrt{\frac{2(r+1)}{1+2r}} \cdot \sigma \quad (2)$$

where r is the anisotropy coefficient.

The equivalent strain can be derived as

$$\varepsilon = \sqrt{\frac{2(r+1)}{1+2r}} \cdot \ln(\rho_0 / \rho) \quad (3)$$

where ρ_0 is the initial location of the material particle at the time $t=0$; ρ is the location of same particle at the time t .

Substituting Eq. (2) into Eq. (1) and considering the strain hardening rule which is expressed by $\sigma=B\varepsilon^n$, we can obtain Eq. (4):

$$d\sigma_\rho = -B \left[\sqrt{\frac{2(r+1)}{1+2r}} \right]^{n+1} [\ln(\rho_0 / \rho)]^n \frac{d\rho}{\rho} \quad (4)$$

Let $B_1 = B \left[\sqrt{2(r+1)/(1+2r)} \right]^{n+1}$. Substituting the boundary condition into Eq. (4), we can obtain the radial stress σ_ρ , which is equal to $\mu Q / (\pi R_w h_0)$ at $\rho=R_w$, where Q is the BHF, h_0 is the initial thickness of the sheet, μ is the friction coefficient, and R_w is the radius of material particle at the outer edge of the flange at the time t . The initial location of the material particle is R_0 , i.e., the radius of the sheet metal. The stress of material particle at any position ρ , where $\rho \in [r_0, R_w]$, and r_0 , the inner radius of the flange, which is the inner radius of the die when the die edge radius is neglected. Then σ_ρ and σ_θ in the flange can be calculated respectively as

$$\sigma_\rho = \int_\rho^{R_w} B_1 \left(\ln \frac{x_0}{x} \right)^n \frac{dx}{x} + \frac{\mu Q}{\pi R_w h_0} \quad (5)$$

$$\sigma_\theta = -B_1 \left(\ln \frac{\rho_0}{\rho} \right)^n + \int_\rho^{R_w} B_1 \left(\ln \frac{x_0}{x} \right)^n \frac{dx}{x} + \frac{\mu Q}{\pi R_w h_0} \quad (6)$$

where $\rho_0 = \sqrt{R^2 - R_w^2 + \rho^2}$; x is the variable of integration, $x \in [\rho, R_w]$; x_0 is the initial location when the time $t=0$; x is the location when the time is t . So x_0 can be expressed as $x_0 = \sqrt{R_0^2 - R_w^2 + x^2}$.

2.2 Inversely proportional assumption

It is difficult to obtain the primitive function of the integral function from Eqs. (5) and (6). So in order to simplify the calculations, some researchers assumed that the equivalent strain is inversely proportional to the radial coordinates. When $\mu=0$, using the equilibrium equations, the Tresca yield criterion and boundary conditions, the stress in the flange can be obtained as [7–11]

$$\sigma = B \left[\sqrt{\frac{2(r+1)}{1+2r}} \ln \frac{R_0}{R_w} \frac{R_w}{\rho} \right]^n \quad (7)$$

$$\sigma_\rho = \frac{B_1}{n} \ln^n \left(\frac{R_0}{R_w} \right) \left[\left(\frac{R_w}{\rho} \right)^n - 1 \right] \quad (8)$$

$$\sigma_{\theta} = \frac{B_1}{n} \ln^n \left(\frac{R_0}{R_w} \right) \left[(1-n) \left(\frac{R_w}{\rho} \right)^n - 1 \right] \quad (9)$$

Figure 1 shows the relationship between the ratio of circumferential stress and intensity coefficient, σ_{θ}/B , and the ratio of radial coordinate and sheet initial radius, ρ/R_0 . Figure 2 demonstrates relation between the equivalent strain, ε , and the ratio of radial coordinate and sheet initial radius, ρ/R_0 , under the two different assumptions when $r=1$, $R_w/R_0=0.85$, $n=0.19$, $r_0=50$ mm, $R_0=110$ mm.

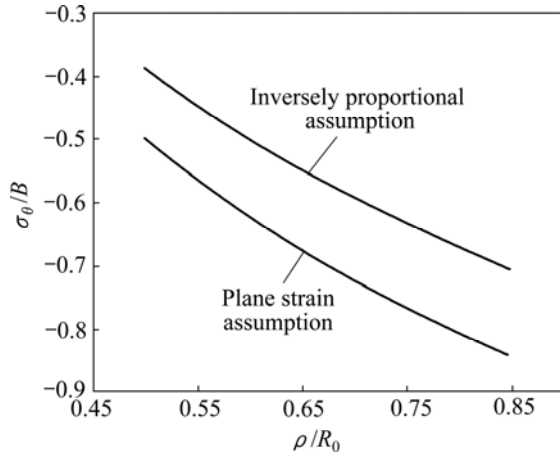


Fig. 1 Ratio of circumferential stress and intensity coefficients under two assumptions

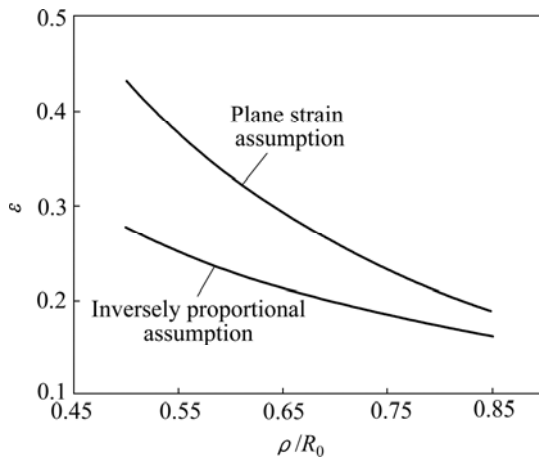


Fig. 2 Equivalent strain under two assumptions

Figures 1 and 2 show that the circumferential stresses are different under the two different assumptions, so are the equivalent strains. The maximum relative error of the circumferential stress is 22.3%, and that of the equivalent strain is greater, which is 35.9%. The circumferential stress and the equivalent strain are needed in order to calculate the BHF when the energy method is used. Hence, the blank holder forces calculated according to the circumferential stress and the equivalent strain under two assumptions are different.

It should be noted that the assumption that the equivalent strain is inversely proportional to the radius is

not in accordance with the constant volume condition. Thus, the stress solution obtained in this way is not accurate. It is also necessary to adopt the later assumption in calculating the BHF, predicting the wrinkling and comparing the results obtained by using the two assumptions.

3 Calculation of strain energy

3.1 Basic equations

Let us consider a single wrinkle. According to the well-known energy method, the energy U_{θ} due to the compressive circumferential stress can be written as

$$U_{\theta} = U_w + U_Q$$

where U_{θ} is strain energy due to circumferential stretching of sheet after wrinkling; U_w is strain energy due to bending stress during wrinkling; U_Q is the work done by the blank holder force.

For a micro-ring with inner radius ρ and outer radius $\rho+d\rho$, its cross section area is $hd\rho$. There are a number of wrinkles. For a single wrinkle of the micro-ring, let l be the original circumferential length, and S' be the increment of circumferential length when the wrinkling happens. Then the strain energy released by the circumferential stress can be obtained.

$$U_{\theta} = - \int_{r_0}^{R_w} S' \sigma_{\theta} t d\rho \quad (10)$$

where $S' = \int_0^l ds - \int_0^l dx$, ds and dx represent the micro circumferential lengths of micro-ring of the single wrinkle after wrinkling and before wrinkling respectively. When the wrinkle magnitude is very small, S' can be expressed by

$$S' \approx \int_0^l \left[1 + \frac{1}{2} \left(\frac{dy}{dx} \right)^2 \right] dx - \int_0^l dx = \frac{1}{2} \int_0^l \left(\frac{dy}{dx} \right)^2 dx$$

Deflection at any point of the single wrinkle is generally assumed to be [7]

$$y = y_0 f_{\phi}(\phi) f_{\rho}(\rho)$$

where y_0 is wrinkle amplitude; ϕ is the angle of arbitrary point in the single wrinkle; y is the deflection of the point at any position (ρ, ϕ) in the flange; $f_{\rho}(\rho)$ and $f_{\phi}(\phi)$ are the functions of ρ and ϕ respectively, which are dimensionless and mutually independent.

Substituting $x = \rho\phi$ into Eq. (11), we can get

$$\frac{dy}{dx} = \frac{\partial y}{\partial x} \frac{d\phi}{dx} = \frac{1}{\rho} \frac{\partial y}{\partial \phi}$$

$$\frac{\partial y}{\partial \phi} = y_0 \frac{df_{\phi}}{d\phi} f_{\rho}(\rho)$$

$$S' = \frac{1}{2} y_0^2 \frac{1}{\rho} [f_\rho(\rho)]^2 \int_0^{\phi_0} \left(\frac{df_\phi}{d\phi}\right)^2 d\phi \quad (11)$$

where ϕ_0 is the angle of the arc corresponding to the single wrinkle. When ρ is an arbitrary value and $\phi=0$ or $\phi=\phi_0$, $y=0$; when $\rho=R_w$ and $\phi=\phi_0/2$, $y=y_0$.

The bending energy that is required to make the wrinkle formed for a single wrinkle can be calculated by

$$U_w = \int_{r_0}^{R_w} \int_0^{\phi_0} \frac{D}{2} \left(\frac{d^2 y}{dx^2}\right)^2 dx dI \quad (12)$$

where dI is the moment of inertia of cross-sectional area of the micro-ring. The inner radius of the ring is ρ . The thickness and the width of the cross section are h and $d\rho$ respectively. And dI can be expressed by

$$dI = \frac{1}{12} t^3 d\rho$$

$$\text{Substituting } \frac{d^2 y}{dx^2} = \frac{1}{\rho^2} \frac{\partial^2 y}{\partial \phi^2} = \frac{1}{\rho^2} \frac{d^2 f_\phi}{d\phi^2} y_0 f_\rho(\rho)$$

into Eq. (12), we have

$$U_w = \int_{r_0}^{R_w} \int_0^{\phi_0} \frac{D}{2} \left(\frac{d^2 f_\phi}{d\phi^2}\right)^2 y_0^2 \frac{[f_\rho(\rho)]^2}{\rho^3} d\phi dI \quad (13)$$

When the swift strain hardening rule is used, we have

$$D = \frac{d\sigma}{d\varepsilon} = nB\varepsilon^{n-1}$$

where D is the plastic tangential modulus.

Let

$$f_\rho(\rho) = \left(\frac{\rho - r_0}{R_w - r_0}\right)^{\frac{1}{2}}$$

$$f_\phi(\phi) = \frac{1}{2} - \frac{1}{2} \cos\left(\frac{2\pi\phi}{\phi_0}\right)$$

Substituting the two equations into Eq. (10) and Eq. (13) respectively, we obtain

$$U_\theta = -\frac{\pi^2}{4\phi_0} \frac{y_0^2}{R_w - r_0} \int_{r_0}^{R_w} \sigma_\theta t \left(1 - \frac{r_0}{\rho}\right) d\rho \quad (14)$$

$$U_w = \frac{1}{12} \frac{\pi^4}{\phi_0^3} \frac{y_0^2}{R_w - r_0} \int_{r_0}^{R_w} D t^3 \frac{(\rho - r_0)}{\rho^3} d\rho \quad (15)$$

In the following analysis, U_θ and U_w are calculated based on the two different assumptions.

3.2 Inversely proportional assumption

Under the condition that the equivalent strain is inversely proportional to the radial coordinate,

substituting the circumferential stress calculated from Eq. (9) into Eq. (14) and let

$$g_\theta(\phi_0) = \frac{B_1 \pi^2 h}{4\phi_0 n} \frac{y_0^2 r_0}{R_w - r_0} \ln^n\left(\frac{R_0}{R_w}\right)$$

The energy released by the circumferential stress σ_θ for a single wrinkle can be written as

$$U_\theta = g_\theta(\phi_0) \left(-\frac{1}{n} + \frac{1}{n} \frac{R_w^n}{r_0^n} - \ln \frac{R_w}{r_0}\right) \quad (16)$$

Because the equivalent strain is inversely proportional to the radial coordinate, we have

$$D = \frac{d\sigma}{d\varepsilon} = nB \left(\frac{R_w}{\rho}\right)^{n-1} \left[\ln\left(\frac{R_0}{R_w}\right) \sqrt{\frac{2(r+1)}{1+2r}}\right]^{n-1} \quad (17)$$

Substituting Eq. (17) into Eq. (15), we can obtain

$$U_w = g_w(\phi_0) \int_{r_0}^{R_w} \left(\frac{R_w}{\rho}\right)^{n-1} \frac{(\rho - r_0) R_w}{\rho^3} d\rho$$

$$\text{where } g_w(\phi_0) = \frac{nB\pi^4 h^3}{12\phi_0^3} \frac{y_0^2}{(R_w - r_0) R_w} \left[\ln\left(\frac{R_0}{R_w}\right) \sqrt{\frac{2(r+1)}{1+2r}}\right]^{n-1}.$$

After the integration, we obtain

$$U_w = g_w(\phi_0) \left[\frac{1}{n} \left(\frac{R_w^n}{r_0^n} - 1\right) - \frac{1}{n+1} \left(\frac{R_w^n}{r_0^n} - \frac{r_0}{R_w}\right)\right] \quad (18)$$

Equations (16) and Eq. (18) are obtained under the condition that the equivalent strain is inversely proportional to the radial coordinate. This condition is contradictory with that of constant volume which is the basic assumption commonly used in the plastic forming.

3.3 Plane strain assumption

Under the condition of plane strain and constant volume, U_θ and U_ϕ are calculated.

In order to simplify and compare the results of the two kinds of calculations, let $\mu=0$. Substituting Eq. (6) into Eq. (14), we can get

$$U_\theta = g_\theta(\phi_0) \int_{r_0}^{R_w} \left(\frac{1}{r_0} - \frac{1}{\rho}\right) \cdot \left\{ \left[\ln(\rho_0/\rho)\right]^n - \int_{r_0/R_0}^{R_w/R_0} \left\{ \ln \left[\sqrt{1 - (R_w/R_0)^2 + x^2} / x \right] \right\}^n / x dx \right\} d\rho \quad (19)$$

where $g_\theta(\phi_0)$ is inversely proportional to ϕ_0 and is calculated by

$$g_\theta(\phi_0) = \frac{B_1 \pi^2 h}{4\phi_0} \frac{y_0^2 r_0}{R_w - r_0}$$

Equation (19) shows that U_θ is the function of R_w and contains many integral terms. Therefore, the

numerical method is generally required to calculate the results.

Using the definition of the equivalent strain and the plane strain assumption, the plastic tangential modulus D can be obtained as

$$D = nB \left(\sqrt{\frac{2(r+1)}{1+2r}} \cdot \ln \frac{\sqrt{R_0^2 - R_w^2 + \rho^2}}{\rho} \right)^{n-1} \quad (20)$$

Substituting Eq. (20) into Eq. (15), we can obtain

$$U_w = g_w(\phi_0) \int_{r_0}^{R_w} \left(\ln \frac{\sqrt{R_0^2 - R_w^2 + \rho^2}}{\rho} \right)^{n-1} \frac{(\rho - r_0) R_w}{\rho^3} d\rho \quad (21)$$

where $g_w(\phi_0)$ is inversely proportional to ϕ_0^3 and is written by

$$g_w(\phi_0) = \frac{nB\pi^4 h^3}{12\phi_0^3} \frac{y_0^2}{(R_w - r_0) R_w} \left[\sqrt{\frac{2(r+1)}{1+2r}} \right]^{n-1}$$

4 Calculation of critical BHF

4.1 BHF under wrinkling condition

From the above analysis, we can get the relationship between the BHF and other parameters by using the energy method based on the latter assumption of the wrinkling model. Let N be the number of the wrinkles, Q be the BHF, then $N = 2\pi/\phi_0$. The work done by the BHF for each wrinkle is obtained.

$$U_Q = \frac{Q}{N} y_0 = \frac{\phi_0 y_0}{2\pi} Q$$

According to the principle of energy conservation, we can obtain

$$Q = (U_\theta - U_w) \frac{2\pi}{\phi_0 y_0} \quad (22)$$

When the friction is not considered in Eq. (22), since Q is not included in U_θ , it can be obtained directly.

When friction is considered, the energy released by the circumferential stress consists of two parts: U_θ and U_θ^μ , where U_θ is the value calculated by Eq. (19) without considering the friction and U_θ^μ is the energy consumption caused by friction force. Substituting the term that contains μ in Eq. (6) into Eq. (10), we can obtain

$$U_\theta^\mu = -K_\mu Q y_0 \frac{\phi_0}{2\pi} \quad (23)$$

where K_μ is dimensionless coefficient and is calculated

$$\text{by } K_\mu = \frac{2\mu}{y_0 R_w \phi_0} \int_{r_0}^{R_w} S' d\rho.$$

Substituting $U_\theta + U_\theta^\mu$ into U_θ in Eq. (22), BHF can be expressed as

$$Q = \frac{1}{1 + K_\mu} (U_\theta - U_w) \frac{2\pi}{\phi_0 y_0}$$

or

$$Q = \frac{2\pi}{\phi_0 y_0 (1 + K_\mu)} \cdot \left\{ g_\theta(\phi_0) \int_{r_0}^{R_w} \left(\frac{1}{r_0} - \frac{1}{\rho} \right) \cdot \left[\left(\ln \frac{\rho_0}{\rho} \right)^n - \int_{\frac{r_0}{R_0}}^{\frac{R_w}{R_0}} \left[\ln \frac{\sqrt{1 - (R_w/R_0)^2 + x^2}}{x} \right]^n \frac{dx}{x} \right] \cdot d\rho - g_w(\phi_0) \cdot \int_{r_0}^{R_w} \left[\ln \frac{\sqrt{R_0^2 - R_w^2 + \rho^2}}{\rho} \right]^{n-1} \frac{(\rho - r_0) R_w}{\rho^3} d\rho \right\} \quad (24)$$

When the assumption that equivalent strain is inversely proportional to the radial coordinate is used, substituting Eq. (16) and Eq. (18) into Eq. (22) and considering friction, Eq. (24) becomes

$$Q = \frac{2\pi}{y_0 \phi_0 (1 + K_\mu)} \left\{ g_\theta(\phi_0) \left(-\frac{1}{n} + \frac{1}{n} \frac{R_w^n}{r_0^n} - \ln \frac{R_w}{r_0} \right) - g_w(\phi_0) \left[\frac{1}{n} \left(\frac{R_w^n}{r_0^n} - 1 \right) + \frac{1}{n+1} \left(\frac{R_w^n}{r_0^n} - \frac{r_0^n}{R_w^n} \right) \right] \right\} \quad (25)$$

When forming conditions, such as forming speed and temperature, are not considered, Eq. (24) and Eq. (25) give the relationship among the BHF and material properties parameters, geometric parameters of the sheet metal, punch stroke and wrinkle model parameters. The BHF changes only with the wrinkling model parameters when the forming conditions and other above parameters remain unchanged.

4.2 Critical BHF

The BHF is the function of the wrinkling model parameters when other parameters have been given. The wrinkle amplitude is generally allowed to be set a certain magnitude under the critical wrinkling condition, and the BHF is only the function of the wrinkle number N or the corresponding central angle of a single wrinkle, ϕ_0 . The critical BHF can be obtained by substituting ϕ_0 into Eq. (24) and Eq. (25). The magnitude of ϕ_0 under the critical BHF condition can be obtained from Eq. (24) and Eq. (25) by using $\partial Q / \partial \phi_0 = 0$.

Figure 3 shows the relation of Q/y_0 vs the number of wrinkles when $R_w/R_0=0.85$ and $\mu=0.1$, where Q is the critical BHF. When the number of wrinkles is a certain value, the BHF reaches the maximum which is the critical BHF.

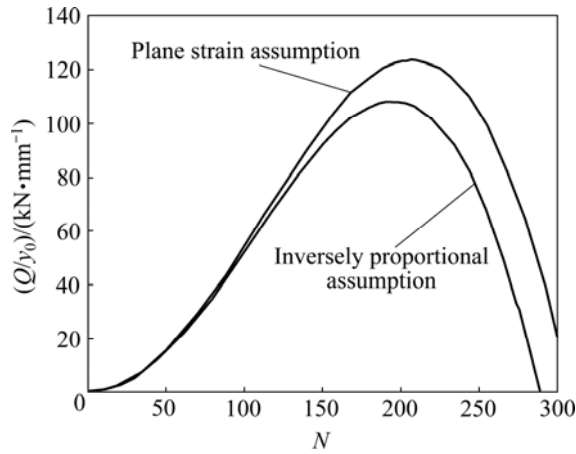


Fig. 3 Q/y_0 vs number (N) of wrinkles ($R_w/R_0=0.85$, $\mu=0.1$)

In the actual drawing process, the critical BHF is the function of the punch stroke in the case that other parameters remain unchanged. The relationship between Q/y_0 and R_w/R_0 is shown in Fig. 4. R_w denotes the position of outer edge of the flange that changes with the punch stroke. The calculated critical BHF under the plane strain assumption is greater than that when the equivalent strain is inversely proportional to the radial coordinate. The critical blank holder forces are both zero at the start and the end positions in the drawing process, while the BHF reaches the maximum at a certain position during the process.

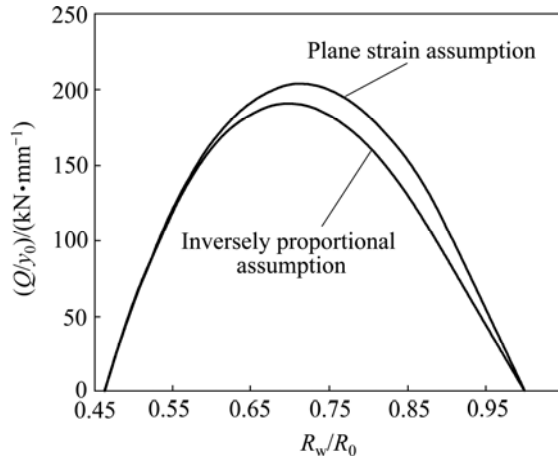


Fig. 4 Curves of critical BHF vs stroke ($\mu=0$)

Generally, wrinkling can always be eliminated in the deep drawing process when the applied BHF is not less than the critical BHF.

5 Influence of wrinkle model on critical BHF

As stated in subsection 3.1, the wrinkle model in flange is generally assumed as

$$y = y_0 f_\phi(\phi) f_\rho(\rho)$$

Referring to the function $f_\rho(\rho)$ used in Eq. (13), it can be revised as

$$f_\rho(\rho) = \left(\frac{\rho - r_0}{R_w - r_0} \right)^m$$

where m is a parameter greater than zero.

Figure 5 demonstrates the shapes of the wrinkle curve along radial direction with different m . When value of m changes, the shapes of the wrinkles are different, while the largest height of the wrinkles is the same. The BHF is mainly applied to the outer edge of the flange where the thickness of the sheet is the greatest after drawing process. The shapes of the wrinkles are convex when $m < 1$, while they are straight or concave when $m = 1$ or $m > 1$ respectively. Generally, the wrinkle shapes are in accordance with experimental result when $m < 1$. Thus the value of m corresponding to the possible shapes of wrinkles in the flange is not greater than 1.0.

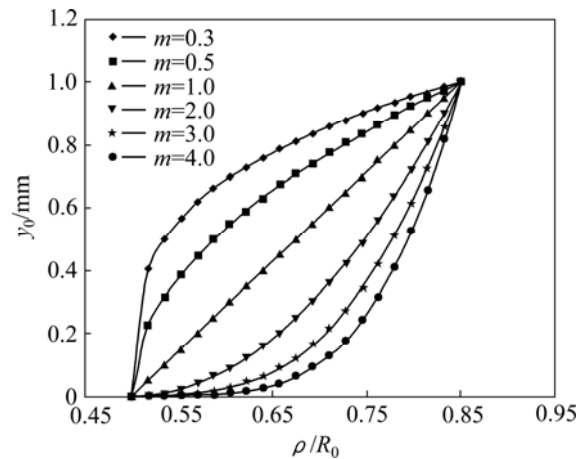


Fig. 5 Shapes of wrinkles along radial direction

Substituting $f_\phi(\phi) = [1 - \cos(2\pi\phi/\phi_0)]/2$ and $f_\rho(\rho)$ for different m into Eq. (10) and Eq. (13), we can get

$$U_\theta = -\frac{\pi^2 y_0^2}{4\phi_0} \int_{r_0}^{R_w} \sigma_\theta h \left(\frac{\rho - r_0}{R_w - r_0} \right)^{2m} \frac{d\rho}{\rho} \quad (26)$$

$$U_w = \frac{\pi^4 y_0^2}{12\phi_0^3} \int_{r_0}^{R_w} D h^3 \left(\frac{\rho - r_0}{R_w - r_0} \right)^{2m} \frac{d\rho}{\rho^3} \quad (27)$$

The strain energy U_θ and U_w can be obtained by substituting Eq. (9) and Eq. (20) into Eq. (26) and Eq. (27) respectively. The critical BHF can be obtained by Eq. (22).

Figure 6 shows the critical BHF curves corresponding to different m . The results show that the shape of the wrinkle has a great effect on the calculated results of the critical BHF. The greater the value of m is, the smaller the critical BHF obtained is.

Under the experimental conditions, there is no

uniform standard to determine the onset of wrinkling, so it is difficult to accurately obtain the critical BHF. Therefore, in many cases, the critical BHF is obtained by analytical or calculation method. Many researchers [7,9–11] selected $m=0.5$ to calculate the critical BHF which is generally greater than the experimental results. The wrinkling occurs in the outer edge of flange at first, and then in the inner range. Thus, the value of m corresponding to the shapes of wrinkles in the flange is possibly not greater than 1. To take a smaller magnitude of m is safe for calculating the critical BHF. The allowed amplitude y_0 and the actual shape of the wrinkles should be considered to calculate the critical BHF accurately.

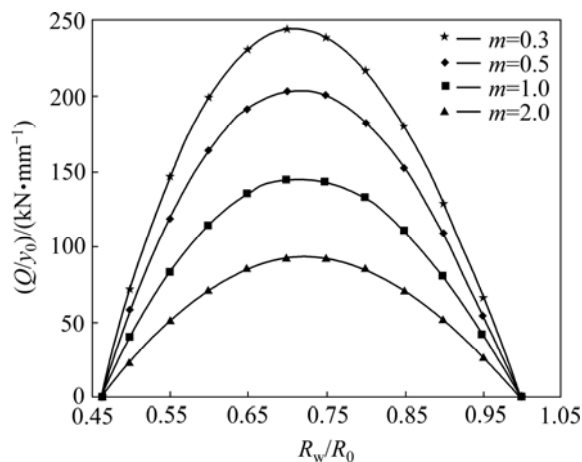


Fig. 6 Relationship between relative critical BHF and shapes of wrinkles (plane strain and $\mu=0$)

In order to reduce the critical BHF in axisymmetric deep drawing process, we can control the wrinkle shapes corresponding to a larger m shown in Fig. 4, in which the critical BHF may be smaller by using a new deep drawing process with radial segmental blank holder, which was proposed by QIN et al [16]. By using this method, the effect of the blank holder can be enhanced.

6 Conclusions

1) The distributions of stress and strain in flange region along the radial direction for axisymmetric deep drawing were analyzed under the plane strain and the condition that the equivalent strain is inversely proportional to the radial coordinate respectively. The maximum relative errors of the circumferential stress and the equivalent strain are 22.3% and 35.9% respectively under the two conditions for some dimensions of sheet and die. The critical blank holding forces are also different.

2) The relationship between BHF and wrinkle number was obtained when wrinkling shape was expressed in terms of power function. The critical BHF under the plane strain was analyzed for the wrinkle

shapes when $m<1$, $m=1$ and $m>1$, respectively. The critical BHF decreases as m increases.

3) The assumed wrinkling shapes have great influences on the critical BHF. Therefore, radial segmental blank holder technique can be utilized to control the wrinkle shapes to improve the formability.

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轴对称拉深成形法兰变形区临界压边力

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摘 要: 提出一种预测临界压边力的数值计算方法。对轴对称拉深成形, 使用能量法分别在平面应变条件和等效应变与位置成反比关系条件下, 给出法兰变形区的周向应力和等效应变与径向坐标的关系。结果表明, 对于定尺寸的板坯和模具, 两种假设条件下的周向应力和等效应变的最大相对误差分别为 22.3%和 35.9%。此外, 在皱纹形状以指数函数表示的条件下, 得到了压边力与皱纹数量的关系。在平面应变假设条件下, 对指数分别小于 1、等于 1 和大于 1 的不同皱纹形状计算了临界压边力。结果表明, 皱纹的假设形状对临界压边力的计算值有较大影响。

关键词: 板材成形; 拉深; 起皱; 临界压边力; 皱纹模型; 皱纹数量

(Edited by LI Xiang-qun)