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Trans. Nonferrous Met. Soc. China 16(2006) 339−345

Transactions of Nonferrous Metals Society of China

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Elastic-plastic solution to stamping thin strip on elastic foundation

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Received 2 June 2005; accepted 16 September 2005

Abstract: An analytical method was proposed to solve the mechanical problems of stamping a thin strip on an elastic foundation. The thin strip was divided into four parts according to its deformation and contact with the punch and the elastic foundation, especially an elastic-plastic part was considered in the deflection of the thin strip. Analytical solutions were derived individually for each part and two models were established with the help of elastic and plastic large deflection theories. Compatibility conditions between the neighboring parts of the thin strip constructed the non-linear equation group. Solutions were carried out by programming with a software. The deformation shape, the membrane force, and the moment and shear force of the deformed thin strip were obtained. The results of the two models were compared. The study shows that the method is effective.

Key words: elastic-plastic solution; beam bending; thin strip stamping; contact mechanics

1 Introduction

In the recent years, there has been growing need for improving quality and decreasing costs of products of the metal forming industry[1−3]. The thin strip metal forming with elastic foundation has been widely used in engineering. A rigid punch presses the thin strip into a pliable polyurethane pad which is displaced, and applies compressive force onto the work-piece surface. This problem is a mechanical problem relating to three-body contact. Contact mechanics has been studied extensively [4−6], but this problem is still challenging because the deformation of the system involves strong non-linearity of geometry and materials.

ZHANG et al[7] and LIN et al[8] studied the problem of stamping a thin strip on elastic foundation, but their studies were only limited to elastic deformation. ZHANG[9] also studied the elastic-plastic deformation of thin strip metals stamped by rigid punch, but the research was only suitable for small deflection. GEIGER et al[10] investigated the bending of U-shaped parts with a rigid punch and an elastic die by use of finite element method, but it required great user-expertise and computational cost. Till present, any analytical solution to the problem has not been very satisfactory, mainly because:

1) The forming process is of large deflection with non-linear material, thus any linear elastic model or small strain elastic-plastic model is not consistent with practice.

2) In analytical approach, the distributions of the interface contact stresses are assumed a priori, which limits the applicability of the solution in terms of the indenter profiles, the ratio of the thickness of the thin strip to punch radius, and the variation of the material properties of the thin strip and elastic foundation.

On the basis of former researches, the authors proposes a new mechanical model which considers large elastic-plastic deflection and non-linearity of materials for the process of thin strip stamped on elastic foundation, and corresponding computation method is performed.

2 Mechanical model

Consider the stamping of a thin strip (thickness: h) on an elastic foundation by a half-circular rigid punch (radius: a). The deformation of the thin strip is in plane

Foundation item: Project(04JJ40035) supported by the Natural Science Foundation of Hunan Province, China; Project(2003AA331090) supported by the High-tech Research and Development Program of China
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stress and is symmetrical to its central line, as shown in Fig.1. The contact zones and interface stresses between the punch and thin strip and the thin strip and foundation are unknown in advance, which are functions of the punch stroke in the stamping process. Because of the symmetry of the deformation, however, we can study half of the thin strip only. In order to analyze the problem properly, we divide the thin strip into four parts as shown in Fig.2:

1) The contact part *AB*, where the thin strip is in perfect contact with both the punch and the foundation surface;

2) The elastic-plastic part *BC*, where the thin strip is in perfect contact with the foundation but has no contact with the punch, its deformation is elastic-plastic;

3) The elastic part *CD*, just like part *BC* but the deformation is only elastic;

4) The free part *DF*, where no contact takes place with either the punch or the foundation.

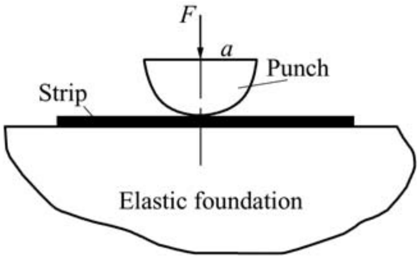

Fig.1 Stamping thin strip on elastic foundation

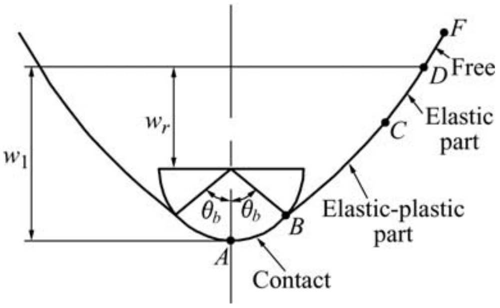

Fig.2 Deformation process and four parts of thin strip

The curvature of the contact part is a known function which is identical to that of the punch outer surface. The contact stresses on the thin strip of this part are the normal contact stresses between the thin strip and punch, Q_{np}, and the normal one between the thin strip and elastic foundation, Q_{ne}. The frictional force is neglected as compared with Q_{np}, Q_{ne}, as shown in Fig.3.

The elastic-plastic part and the elastic part can be modelled as a cantilever beam subjected to normal stress Q_{ne}, due to the interaction between the thin strip and foundation as shown in Fig.4. The end *B* of this part is the contact-off point between the punch and the thin strip, and the end *D* is the contact-off point between the thin strip and the elastic foundation. The boundary conditions of the cantilever must be specified to guarantee the continuity of stresses and deformation across these two ends, i.e.

1) The bending moment, membrane and shear force are zero at point *D*.

2) The deflection and its slope, bending moment and membrane force are equal to those of the contact part *AB* at *B*.

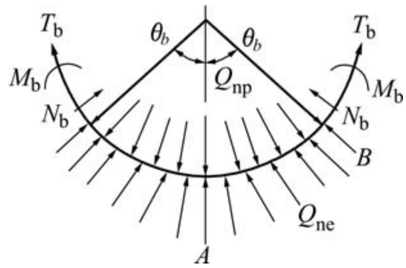

Fig.3 Mechanical model for contact part *AB*

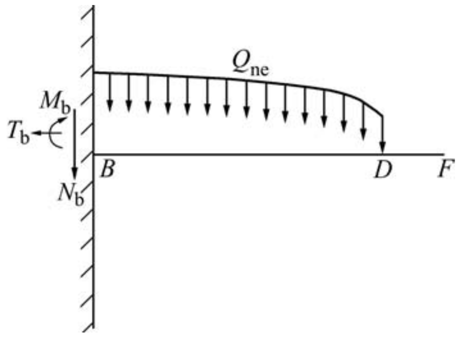

Fig.4 Mechanical model for parts *BC*, *CD* and *DF*

The free part of the thin strip, *DF*, does not deform during the stamping process. Its displacement relies on the deflection and deflection slope at end point *D*, and can be calculated easily when the solution to *CD* is obtained. Hence, we will ignore this part in the following analysis.

To simplify the computation of the contact stresses, we assume that the normal reaction of the elastic foundation follows the Winkler's hypotheses.

2.1 General mechanical description

It is postulated that the thin strip metals can be characterized by the ideal elastic-plastic model[6]. The elastica of the thin strip bending of large deflection can be expressed as[11]

$$\frac{dX}{dS}=\cos\theta,\ \frac{dY}{dS}=\sin\theta,\ \frac{d\theta}{dS}=\frac{M}{EI}=\frac{1}{R},\quad M/M_e\leqslant 1 \qquad (1)$$

The plastica of the thin strip bending of large deformation can be expressed as[12]

$$\frac{dX}{dS}=\cos\theta,\ \frac{dY}{dS}=\sin\theta,\ \frac{d\theta}{dS}=\frac{M_e/EI}{\sqrt{3-2M/M_e}}=\frac{1}{R},$$

$$1\leqslant M/M_e\leqslant\frac{3}{2} \tag{2}$$

where X and Y are Cartesian coordinates, θ is the tangent angle of the deformed thin strip surface with the positive direction of X-axis(Fig.4), dS is the length of an infinitesimal thin strip element(Fig.4), E is the elastic modulus of the thin strip material, I is the second moment of area per unit width of the thin strip cross section, M is the bending moment of the thin strip, R is the curvature radius of the thin strip, and M_e is the bending moment of elastic limit. The equilibrium equation of the thin strip with large deflection can be written as[13]

$$\frac{dT}{d\phi}+N=0,\quad \frac{dN}{d\phi}-T+RQ_n=0,\quad \frac{dM}{d\phi}-RN=0 \tag{3}$$

where T is the membrane force of the thin strip, N is the shear force of the thin strip, Q_n is the normal stress, $Q_n=Q_{np}-Q_{ne}$, and ϕ is the included angle of the external normal of the deformed thin strip surface at θ with the Y-axis.

Above Eqns.(1)-(3) can also be written as non-dimensional forms to simply the solutions as follows:

$$\frac{dx}{ds}=\cos\theta,\quad \frac{dy}{ds}=\sin\theta,\quad \frac{d\theta}{ds}=\eta_1 m=\frac{1}{r} \tag{4}$$

$$\frac{dx}{ds}=\cos\theta,\quad \frac{dy}{ds}=\sin\theta,\quad \frac{d\theta}{ds}=\frac{\eta_1}{\sqrt{3-2m}}=\frac{1}{r} \tag{5}$$

$$\frac{dt}{d\phi}+n=0,\ \frac{dn}{d\phi}-t+vr(q_{np}-q_{ne})=0,$$

$$\frac{dm}{d\phi}-6vrn=0 \tag{6}$$

$$q_{ne}=\eta_2 w \tag{7}$$

where

$$x=\frac{X}{a},\quad y=\frac{Y}{a},\quad s=\frac{S}{a},\quad r=\frac{R}{a},\quad w=\frac{W}{a},$$

$$M_e=\frac{1}{6}\sigma_y h^2,\quad N_e=\sigma_y h,\quad T_e=\sigma_y h,$$

$$m=\frac{M}{M_e},\quad n=\frac{N}{N_e},\quad t=\frac{T}{T_e},\quad q_{ne}=\frac{Q_{ne}}{\sigma_y},$$

$$q_{np}=\frac{Q_{np}}{\sigma_y},\quad v=\frac{a}{h},$$

$$\eta_1=\frac{2\sigma_y a}{Eh},\quad \eta_2=\frac{E_d}{2\sigma_y}$$

where σ_y is the yield stress of the thin strip, and E_d is the elastic modulus of the elastic foundation.

2.2 Solution to part *AB*

The deformation of the thin strip in this zone exactly follows the geometrical profile of the punch, and its deformation is plastic. Using the geometrical equation of a circle, we have

$$x=\sin\theta,\ y=-\cos\theta,\ r=1 \tag{8}$$

$$w=w_r+\cos\theta \tag{9}$$

where w is the perpendicular displacement at any point of part AB, w_r is the punch displacement at the punch center(Fig.2). The force in the thin strip can easily be determined when Eqns.(8) and (9) are substituted into Eqns.(6) and (7) and noticing $\phi=\theta$.

$$m=\frac{3-\eta_1^2}{2},\quad n=0,\quad t=t_0 \tag{10}$$

where t_0 is a dimensionless membrane force at the central point A of the thin strip. If t_0 is known, the normal contact stress between the thin strip and the punch is

$$q_{np}=\eta_2(w_r+\cos\theta)+\frac{t_0}{v} \tag{11}$$

2.3 Solution to part *BC*

When a force (or several forces) acts on a beam, the produced bending moment can always be written as the linear function of the coordinates x and y[12], we use this relation here.

$$m=Ax+By+C \tag{12}$$

where A, B and C are different constants, which are related to the quantity and position of the forces.

Using the third equation of Eqn.(5), we have

$$\frac{ds}{d\theta}=\frac{1}{\eta_1}\sqrt{3-2m}$$

Thus

$$\frac{d}{d\theta}\left(\frac{ds}{d\theta}\right)=-\frac{1}{\eta_1\sqrt{3-2m}}\cdot\frac{dm}{d\theta}=-\frac{1}{\eta_1^2}\cdot\frac{d\theta}{ds}\cdot\frac{dm}{d\theta}=$$

$$-\frac{1}{\eta_1^2}\cdot\frac{dm}{ds} \tag{13}$$

Substituting of Eqn.(12) into Eqn.(13), and then using the first two equations of Eqn,(5), yield

$$s=\frac{A}{\eta_1^2}\cos\theta+\frac{B}{\eta_1^2}\sin\theta+C_1\theta+C_2 \tag{14}$$

where C_1 and C_2 are integration constants.

Substituting Eqn.(14) into the first two equations of Eqn.(5), thus we obtain the thin strip equation:

$$x=\frac{A}{4\eta_1^2}\cos 2\theta+\frac{B}{4\eta_1^2}(2\theta+\sin 2\theta)+C_1\sin\theta+C_3 \tag{15}$$

$$y=-\frac{A}{4\eta_1^2}(2\theta-\sin 2\theta)-\frac{B}{4\eta_1^2}\cos\theta-C_1\cos\theta+C_4 \tag{16}$$

where C_3 and C_4 are also integration constants.

According to the third equation of Eqn.(6), we have the shear force of the thin strip in part BC,

$$n=\frac{1}{6\nu r}\frac{\mathrm{d}m}{\mathrm{d}\phi}=\frac{1}{6\nu}\frac{\mathrm{d}m}{\mathrm{d}s}=\frac{1}{6\nu}(A\cos\theta+B\sin\theta) \tag{17}$$

Using the second equation of Eqn.(6), the membrane force of the thin strip in part BC is,

$$t=\frac{1}{6\nu}(-A\sin\theta+B\cos\theta)-$$
$$\nu\eta_2(w_r-y)(-\frac{A}{\eta_1^2}\sin\theta+\frac{B}{\eta_1}\cos\theta+C_1) \tag{18}$$

2.4 Solution 1 to part *CD*

Only elastic deformation appears in this part, the bending deflection of this part is far smaller than its length. Even though the thin strip has deformed, it can approximately be seen as a straight line. Hence, the coordinate x of the deformed thin strip is proportional to its coordinate y. Thus according to Eqn.(12), the produced moment of the thin strip in this part can be written as

$$m=gy+p \tag{19}$$

where g and p are constants. We use this expression for distributed load.

Substitution of Eqn.(19) into the third equation of Eqn.(4), yields

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}s^2}=\eta_1 g\sin\theta$$

Notice

$$\frac{\mathrm{d}}{\mathrm{d}s}[(\frac{\mathrm{d}\theta}{\mathrm{d}s})^2]=2\frac{\mathrm{d}\theta}{\mathrm{d}s}\cdot\frac{\mathrm{d}^2\theta}{\mathrm{d}s^2}$$

Thus

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}s^2}=\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}\theta}[(\frac{\mathrm{d}\theta}{\mathrm{d}s})^2]=\eta_1 g\sin\theta$$

Integrating this equation, and noticing $C_5=2\eta_1 g\cdot\cos\theta_\mathrm{d}$, we have

$$\mathrm{d}s=\frac{1}{\sqrt{-2\eta_1 g}}\frac{\mathrm{d}\theta}{\sqrt{\cos\theta-\cos\theta_\mathrm{d}}} \tag{20}$$

Assuming

$$k=\sin\frac{\theta_\mathrm{d}}{2},\quad \sin\phi=\sin\frac{\theta}{2}/k$$

then yields

$$\mathrm{d}s=\frac{1}{\sqrt{-\eta_1 g}}\frac{\mathrm{d}\phi}{\sqrt{1-k^2\sin^2\phi}} \tag{21}$$

In point C, $\phi_c=\arcsin(\sin(\theta_c/2)/k)$. In point D, $\phi_d=\pi/2$. Thus integration of Eqn.(21) may obtain the arc length of part CD,

$$S_{CD}=\frac{1}{\sqrt{-\eta_1 g}}[F(k,\frac{\pi}{2})-F(k,\phi_c)] \tag{22}$$

In Eqn.(22),

$$F(k,\frac{\pi}{2})=\int_0^{\frac{\pi}{2}}\frac{\mathrm{d}\phi}{\sqrt{1-k^2\sin^2\phi}}$$

is called the complete elliptical integration of the first kind; and

$$F(k,\phi_c)=\int_0^{\phi_c}\frac{\mathrm{d}\phi}{\sqrt{1-k^2\sin^2\phi}}$$

is called the incomplete elliptical integration of the first kind. Substitution of Eqn.(21) into the first two equations of Eqn.(4), yields

$$x=x_c+\frac{1}{\sqrt{-\eta_1 g}}[2E(k,\phi)-2E(k,\phi_c)-$$
$$F(k,\phi)+F(k,\phi_c)] \tag{23}$$

$$y=y_c+\frac{2k}{\sqrt{-\eta_1 g}}(\cos\phi_c-\cos\phi) \tag{24}$$

where x_c and y_c are coordinates of point C in terms of Eqns.(15) and (16), and

$$E(k,\phi)=\int_0^{\phi}\sqrt{1-k^2\sin^2\phi}\,\mathrm{d}\phi$$

is called the incomplete elliptical integration of second kind.

In point D, $\phi_d=\pi/2$, and its longitudinal coordinate $y_d=w_r$. According to Eqn.(24), we have

$$w_r=y_c+\frac{2k}{\sqrt{-g\eta_1}}\cos\phi_c \tag{25}$$

Because D is a free point, no drawing force acts on it, and the pressure of the elastic foundation is perpendicular to the thin strip, thus the membrane force of the thin strip in part *CD* in fact is zero. That is

$$t=0 \tag{26}$$

The moment in point *D* is zero, and the longitudinal coordinate $y_d=w_r$. Substitution of these conditions into Eqn.(19) may attain the moment of the thin strip in part *CD*,

$$m=-g(w_r-y) \tag{27}$$

According to the third equation of Eqn.(5), we have the shear force of the thin strip in part *CD*,

$$n=\frac{1}{6v}\frac{\mathrm{d}m}{\mathrm{d}s}=\frac{gk}{3v}\sin\phi\sqrt{1-k^2\sin^2\phi} \tag{28}$$

2.5 Solution 2 to part *CD*

The numerical results of the solution 1 to part *CD* show that the elastic part *CD* is very straight. Experience tells us that the elastic part is also straight. So as to simplify the computation, and to avoid special functions, we further assume the elastic part *CD* is a straight line. That is to say, the tangent angle of the thin strip in part *CD* $\theta=\theta_c=\theta_d$. Thus according to the first two equations of the Elastica 4, we may obtain the coordinates of the part *CD*,

$$x=x_c+s\cos\theta_c \tag{29}$$

$$y=y_c+s\sin\theta_c \tag{30}$$

where s is the arc length of the thin strip in part *CD* from the beginning of point *C*.

The moment of the thin strip in part *CD* is still the Eqn.(27). Because the point *C* is a connective point from elastic state into plastic state, thus the moment of the thin strip in this point is the elastic limited moment. So when $y=y_c$, we have $m_c^+=1$. Substituting this condition into Eqn.(19), we obtain the moment of the thin strip in part *CD*

$$m=\frac{w_r-y}{w_r-y_c} \tag{31}$$

And in terms of Eqn.(28), the shear force of the thin strip in the part *CD* is

$$n=\frac{-\sin\theta_c}{6v(w_r-y_c)} \tag{32}$$

The membrane force of the thin strip in part *CD* is zero, too.

3 Numerical method

Above analyses may be carried out by programming, but the constants which appear in the above equations should be determined at first. They may be decided by the compatibility conditions of the parts of the thin strip.

3.1 Numerical model 1

This model uses the solution 1 of the part *CD*. In the point *B* of part *AB*, the tangent angle $\theta=\theta_b$. According to Eqns.(9) and (11), we have the expressions of x_b^-, y_b^-, t_b^-, n_b^-, m_b^-.

In the point *B* of part *BC*, also $\theta=\theta_b$. According to Eqns.(12), (15)-(18), we have the expressions of x_b^+, y_b^+, t_b^+, n_b^+, m_b^+.

In the point *C* of part *BC*, the tangent angle $\theta=\theta_c$. According to Eqns.(12), (16)-(18), we have the expressions of y_c^-, t_c^-, n_c^-, m_c^-.

In the point *C* of part *CD*, also $\theta=\theta_c$, $\varphi_c=\arcsin(\sin(\theta_c/2)/k)$.

Using the solution 1 of part *CD*, that is in terms of Eqns.(26)-(28), we have the expressions of t_c^+, m_c^+, n_c^+.

According to Eqns.(23) and (24), the compatibility of coordinates in the point *C* of part *BC* and part *CD* is satisfied, but according to Eqn.(25), we can get another equation of y_c^+.

From above expressions, we may construct 10 non-linear equations as listed in Table 1. In the 10 non-linear equations, there are 11 unknown parameters, so it is necessary to add another equation. From Eqn.(11) we know that at $\theta=0$, the normal contact stress q_{np} between the thin strip and the punch is maximal. It will affect the plastic forming of the thin strip, and results in the contact part *AB* being in plastic state. So we assume that when $\theta=0$, $q_{\mathrm{np}}=1$.Thus there are 10 unknowns in the 10 non-linear equations in Table 1. The problem may be solved.

Table 1 Equations and unknowns of numerical model 1

Equation	Expression	Unknown
y_1	$x_b^-=x_b^+$	θ_b, A, B, C_1, C_3
y_2	$y_b^+=y_b^-$	θ_b, A, B, C_1, C_4
y_3	$t_b^+=t_b^-$	t_0, A, B, θ_b, C_1
y_4	$n_b^+=n_b^-$	A, B, θ_b
y_5	$m_b^+=m_b^-$	A, B, C
y_6	$y_c^+=y_c^-$	$k(\theta_d), \theta_c(\phi_c), A, B, C_1, C_3$
y_7	$t_c^-=0$	A, B, θ_c, C_1
y_8	$n_c^+=n_c^-$	$k(\theta_d), \theta_c(\phi_c), A, B, g$
y_9	$m_c^-=1$	A, B, C
y_{10}	$m_c^+=1$	g

3.2 Numerical model 2

This model uses the solution 2 of the part *CD*. Part equations in the above numerical model 1 are still valid here. According to the solution 2 to part *CD*, we can obtain the additional boundary conditions in the point C of part *CD* are t_c^+, n_c^+. Thus we can attain 8 non-linear equations as listed in Table 2. Unknowns in the equations are just 8, that is $\theta_b, \theta_c, A, B, C, C_1, C_3, C_4$. The model can be solved.

Table 2 Equations and unknowns of numerical model 2

Equation	Expression	Unknown
y_1	$x_b^- = x_b^+$	θ_b, A, B, C_1, C_3
y_2	$y_b^+ = y_b^-$	θ_b, A, B, C_1, C_4
y_3	$t_b^+ = t_b^-$	A, B, θ_b, C_1
y_4	$n_b^+ = 0$	A, B, θ_b
y_5	$m_b^+ = m_b^-$	A, B, C
y_6	$t_c^- = 0$	A, B, θ_c, C_1, C_4
y_7	$n_c^+ = n_c^-$	θ_c, A, B, C_1, C_4
y_8	$m_c^- = 1$	A, B, C

4 Numerical results

The proposed models may be accomplished in Matlab software with programming. The non-linear equations may be solved by a least squares method, and the numerical integral is calculated by the adaptive Simpson quadrature[14]. As an numerical example, the thin strip is a low carbon steel of No.15, its yield stress σ_y=226 MPa, and its elastic modulus E=200 GPa; the elastic foundation is a kind of polyurethane 80 A, its elastic modulus E_d=30 MPa. The punch radius a=50 mm. The thin strip thickness h=3.0 mm. The computed results are shown in Figs.5-10 at the punch stroke w_r=1.

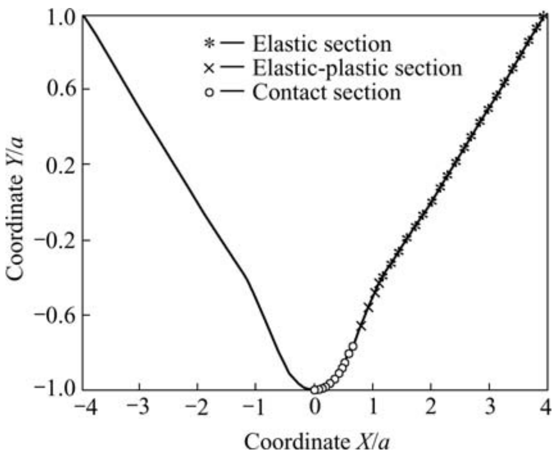

Fig.5 Deflection process and shape of thin strip of model 1

Figs.5 and 6 show the deflective shapes of the thin strip of model 1 and model 2 when the punch stroke is 1. It can be seen that the deflective shape of the thin strip just likes a V word. They obviously have three parts, including the contact part, the elastic-plastic part and the elastic part. Furthermore, the length of the elastic-plastic part is not very short, it can not be neglected. It is clear that the elastic part is very straight, which matches the case of practice. The span of the V-shape is related to the elastic modulus of the elastic foundation. The difference

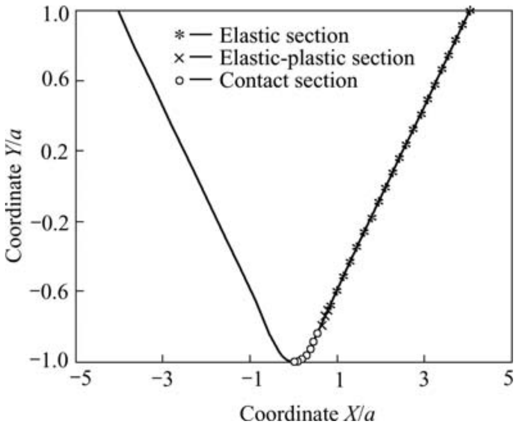

Fig.6 Deflection process and shape of thin strip of model 2

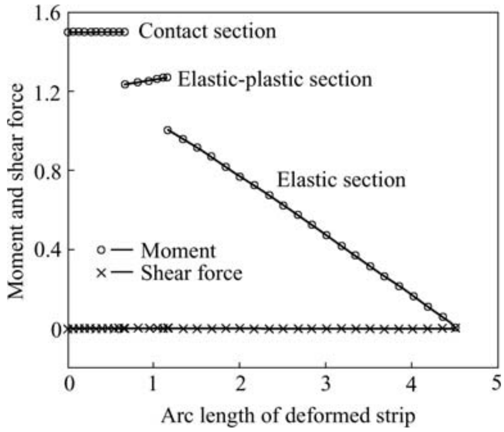

Fig.7 Moment and shear force of deformed thin strip of model 1

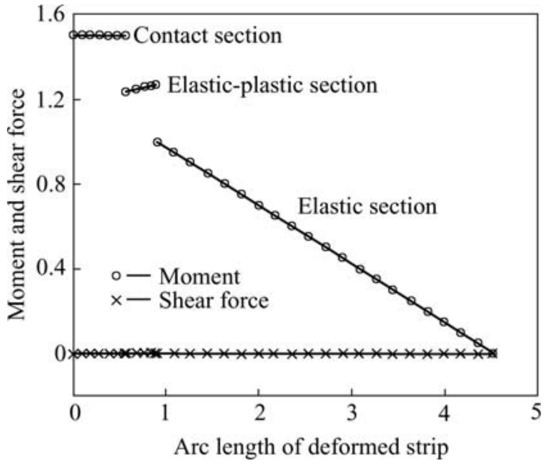

Fig.8 Moment and shear force of deformed thin strip of model 2

of model 1 and model 2 reflected to the deflection of the thin strip is that the deflection of the thin strip is smooth in model 2, but not smooth and has a inflexion in model 1. That is to say, model 2 seems to be more reasonable.

Figs.7 and 8 show the moments and shear forces of the deformed thin strip. The curve for shear force is ideal. Theoretically, the shear force for contact part AB should exactly be zero from Eqn.(11), and at the end point D of the elastic part it is also zero since DF part is free. These are all satisfied in the computed curve. This curve also shows that the shear force in the elastic-plastic part and the elastic part is very small.

The moment of the contact part *AB* is a constant, and the moment of the elastic part *CD* is a linear function of the coordinates. These are all met. The moment of the elastic-plastic part *BC* should satisfy the boundary conditions at both points *B* and *C*, but computation shows they are very difficult to meet. To speak exactly, the convergent precision of the equations y_5 and y_9 in model 1 and y_5 and y_8 in model 2 are not very good. In the optimal processes of solving the non-linear equations, we already have magnified these equations 10 times. If they are magnified more, it will deteriorate the convergent precision of other equation. The discontinuity error of the moment in both points *B* and *C* is about 20%. The cause resulting in this instance seems to be that Eqn.(12) is not strict under the distributed load.

The membrane forces of the thin strip are shown in Figs.9 and 10. The curves are continuous. The membrane force of the contact part *AB* is a constant, and it is zero in the elastic part *CD*. The curves for the elastic-plastic part are smooth. The numerical results are good. The difference between model 1 and 2 is that the arc length of corresponding part is different. In model 1, the arc length of the contact part is about 0.6, the elastic-plastic part is about 0.63. In model 2, the arc length of the contact part is about 0.6, the elastic-plastic part is about 0.35. The cause resulting in this instance is because the convergent precision of the two models is different. Since the convergent precision of model 2 is 10^{-4}, and better than model 1, the results of model 2 are believable and more accurate.

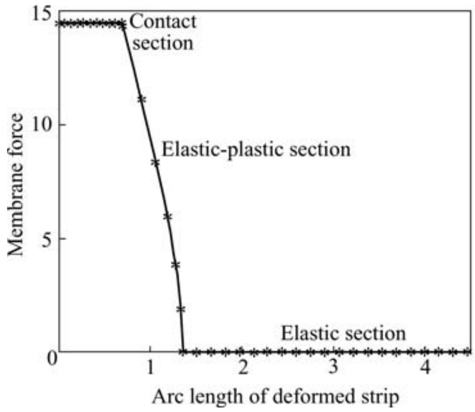

Fig.9 Membrane force of deformed thin strip of model 1

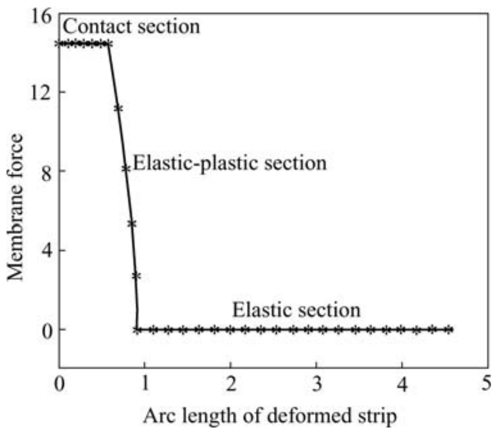

Fig.10 Membrane force of deformed thin strip of model 2

5 Conclusions

An analytical method is proposed to solve the mechanics problem of stamping a thin strip on an elastic foundation. Two models are established with the help of elastic-plastic theory of large deformation. Especially the elastic-plastic part is considered in the deflection of the thin strip. The results of the two models are compared. The result of the simplified model is more satisfactory. The method adopted in this paper may be extended to consider clad thin strip's stamping.

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(Edited by YUAN Sai-qian)