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Trans. Nonferrous Met. Soc. China 21(2011) s642-s647

Transactions of Nonferrous Metals Society of China

www.tnmsc.cn

Endemic error correction model and quantitative analysis of precise point positioning

JI Chang-dong, FENG Lei, XU Ai-gong

School of Geomatics, Liaoning Technical University, Fuxin 123000, China

Received 19 June 2011; accepted 10 November 2011

Abstract: To identify the endemic error of the precise point positioning which cannot be weakened or eliminated in precise point positioning (PPP) zero-difference model, the 24 h observation data acquired from CHAN station on Oct 31st, 2010, were adopted for analyses, different correction models of various errors were discussed and their influences on traditional zero-difference model were analyzed. The results show that the errors cannot be ignored. They must be corrected with suitable models and estimated with auxiliary parameters. The influence magnitudes of all errors are defined, and the results have guiding significance to improve the accuracy of precise point positioning zero-difference model.

Key words: precise point positioning (PPP); zero-difference model; endemic error; GPS qualitative analysis

1 Introduction

In 1997, ZUMBERGE et al [1] in the America Jet Propulsion Laboratory (JPL) proposed the concept of precise point positioning (PPP). In the late 1990s, JPL, as one of the IGS data analyses centers, began to take this method to process the data from non-nuclear GPS stations [2]. The major error sources of GPS absolute positioning include orbit error, satellite clock bias, and delay. If using dual-frequency ionospheric and dual-stream receiver, the influence of ionospheric delay can be eliminated with LC phase combination. If earth-centered and earth fixed coordinates are chosen and the calculation reference framework is the same, the earth rotation parameter in the observation equation can be eliminated [3–4]. So, if the satellite orbit and precise clock bias are fixed and precise observation model is used, we can identify the exact location of the receiver, clock bias, ambiguity, and tropospheric delay parameter of a single station just as we can work out the pseudo range [5–6]. This is the basic ideas of precise point positioning. Since there is no direct way to eliminate or weaken the same or similar errors by differentiation method, errors in precise point positioning can be corrected precisely by models or estimated as undetermined parameters.

2 Mathematical model of precise point positioning

Traditional PPP model adopts dual-frequency GPS pseudo range and ionospheric-free combination of carrier phase observation to form the observation model so as to weaken the influence of ionosphere. Both ZUMBERGE et al [1] and KOUBA and HEROUX [3] applied this combination as the functional model for precise point positions. The observation model is expressed as follows:

$$P_{\rm IF} = \frac{f_1^2}{f_1^2 - f_2^2} P_1 - \frac{f_2^2}{f_1^2 - f_2^2} P_2 = \rho + cdt + d_{\rm trop} + \varepsilon(P_{\rm IF})$$
(1)

$$\Phi_{\rm IF} = \frac{f_1^2}{f_1^2 - f_2^2} \Phi_1 - \frac{f_2^2}{f_1^2 - f_2^2} \Phi_2
= \rho + cdt + d_{\rm trop} + B_{\rm IF} + \varepsilon(\Phi_{\rm IF})$$
(2)

where $P_{\rm IF}$ represents the ionopheric-free pseudo range combination observation; P_1 and P_2 represent the ionosphere-free pseudo range combination observations respectively; $\Phi_{\rm IF}$ represents the ionosphere-free phase combination observation (distance) of Φ_1 and Φ_2 ;

Foundation item: Project (20060417004) supported by the PhD Programs Foundation of Ministry of Education of China; Project (2009S049) supported by the Liaoning Province University Research Program, China

Corresponding author: JI Chang-dong; Tel: +86-418-3351817; E-mail: wn1529@163.com

 $f_i(i=1, 2)$ represents the carrier frequency; $B_{\rm IF} = \frac{f_1^2}{f_1^2 - f_2^2} \lambda_1 N_1 - \frac{f_2^2}{f_1^2 - f_2^2} \lambda_2 N_2$ represents the ambiguity

of ionosphere-free phase combination observation; $\varepsilon(P_{\rm IF})$ and $\varepsilon(\Phi_{\rm FF})$ represent the observation noise and non-modeled error of the two combination observations, respectively. The combination ambiguity of ionosphere-free phase observation does not possess the characteristics of an integer. Meanwhile, the observation noise of the combination observation is magnified, which results in a 30 min convergence time of precise point positioning.

3 Endemic errors of precise point positioning

Generally, the errors of GPS positioning are divided into several categories, errors related to the GPS satellite, errors related to the propagation path, and errors related to the receivers[4]. However, specific to precise point positioning, in addition to satellite orbit error, satellite clock bias, ionospheric delay error, tropospheric delay, receiver's clock bias, multi-path error, and observation noise which should be considered, errors caused by satellite and receiver's antenna phase center deviations, phase wrapping, relativistic effect, solid tide, marine load, atmospheric load, and earth rotation should also be taken into account. These errors are called endemic errors of precise point positioning. Only correcting those errors mentioned above with precise models can we keep positioning accuracy [5–9].

4 Model corrections and quantitative analysis for endemic error in precise point positioning

To make comparison of the influence magnitudes of all errors, the factors which are taken into account by both traditional error and endemic error are called full models. The major factors include satellite clock bias correction, satellite motion in the signal transmission process, earth rotation in the signal transmission process, satellite centroid to antenna phase center correction, receiver's antenna phase center and reference point correction, relativity correction, tropospheric delay (Niell projection function is in use), P1-C1 differential code biases (DCB) correction, phase wrapping and solid tide correction, and etc. The three-axis directional error of full model precise point positioning is expressed in Fig. 1. To analyze certain error, it is just needed to neglect this error.

4.1 Earth rotation correction in signal transmission process

Since earth-fixed coordinate system does not belong



Fig. 1 Full model of three-axis directional error

to the inertial coordinate system, it revolves when the earth rotates. So, the corresponding earth-fixed coordinate systems specific to the time of satellite signal transmission and receiver's signal receiving time are different from each other. Just as shown in Fig. 2, its influence magnitude in the east is about 25 m. Suppose the observation station coordinate is (x_R, y_R, z_R) , the satellite coordinate is (x_S, y_S, z_S) , σ is the earth spin velocity and *c* is the light velocity in vacuum, then, the layback correction caused by earth rotation is as follows[10]:

$$\Delta D_{\overline{\sigma}} = \frac{\overline{\sigma}}{c} [y_{\rm S}(x_{\rm R} - x_{\rm S}) - x_{\rm S}(x_{\rm R} - x_{\rm S})]$$
(3)



Fig. 2 Non-earth-rotation correction of three-axis directional error

4.2 Satellite center to antenna phase center correction

Satellite orbit data provided by broadcast ephemeris and precise ephemeris are based on different data. Especially, the satellite position provided by SP3 format is related to the satellite centroid. Satellite mass center generally does not coincide with the phase center of satellite transmitting antenna, which is called antenna phase center error. The observation value is from receiver's antenna phase center to satellite antenna phase center [11]. The deviation between the two in satellite-fixed coordinate system is listed in Table 1. It can be acquired through ANTEX file and corrected as expressed in Eq. (4). As shown in Fig. 3, the error is about 0.1 m and it is related to the type of the antenna.

$$X_{\text{phase}} = X_{\text{mass}} + \begin{bmatrix} e_x & e_y & e_z \end{bmatrix}^{-1} X_{\text{offset}}$$
(4)

In Eq. (4), (e_x, e_y, e_z) represent the unit factors of satellite-fixed coordinate system axes in the inertial coordinate system, respectively; $(X_{\text{phase}}, X_{\text{mass}})$ represents the phase-centered satellite coordinate in the inertial coordinate system and weight-centered satellite coordinate, respectively; X_{offset} represents the deviation of satellite antenna phase center in the satellite-fixed coordinate system.

 Table 1
 Satellite antenna phase center deviation in satellite-fixed coordinate system

Satellite type	<i>x</i> /m	y/m	z/m
Block II/IIA	0.279	0	1.023
Block IIR	0	0	0



Fig. 3 Three-axis directional error correction from non-satellite centroid to phase center

4.3 Receiver's antenna phase center and reference point correction

Generally speaking, the location of observation station is in relative to the term observation station base. The location of this base is different from the antenna phase center. The error caused by it must be taken into consideration and it is closely related with the frequency of carrier wave [12], as shown in Fig. 4. It is about 0.3 m in magnitude. Besides, the location of observation station is also relative to other specific positions (e.g., the geodetic point of the earth surface), and it can be corrected with three components, north, east and up errors.



Fig. 4 Three-axis error correction of non-receiver antenna phase center

4.4 Relativity effect correction

Both GPS satellite and land receiver have clocks. According to special relativity and general relativity principles, satellite and receiver have different traveling speeds and gravity potentials in inertial space. The influence magnitude of relativity effect is around 6 m, as shown in Fig. 5.



Fig. 5 Three-axis directional error in non-relativistic correction

The processing procedure of special relativity's influence is as follows [13].

Firstly, taking user and GPS satellite orbit as circular orbit, the influence at this time is a normal component, which is called secular term effect. It can be dealt with by adjusting the frequency of satellite clock oscillator. So, the frequency modulation signal frequency from GPS satellite is 10.22999999545 MHz, but not 10.23 MHz.

Secondly, GPS orbit is actually an ellipse in shape. Its influence is a periodic term. It shows a periodic change in pace with the satellite motion. With the elapse of time, its linearity increases which can be dealt with by adjusting user's receiver and its influence magnitude can reach 13 m.

The influence of general relativity's observation value is related to the height of satellite, which is about 4 cm.

4.5 P1–C1 DCB correction

DCB is the deviation generated by the influence from electronic device [14], antenna, receiver, and cable of the transmission equipments directly on the observation value. Since the generation of the code is related to the receiver type, it must be corrected by relative information of the receiver. Either P_1 or P_2 is suitable for this correction. The influence of this error is small in magnitude, as shown in Fig. 6.



Fig. 6 Three-axis directional error in non-P1-C1 DCB correction

4.6 Phase wrapping correction

GPS satellite signal is right-handed polarized wireless signal. The carrier phase observation depends on mutual turning of the satellite and receiver antenna. Generally speaking, a receiver does not turn around automatically. But the antenna of the satellite will turn very slowly as the direction of solar panel towards the sun varies. When it revolves for a complete circle around the central axis, the carrier phase varies for a complete circle. Correspondingly, the geometrical relationships between the satellite and the observation station change. Its influence on zero-difference model should not be neglected. The influence of the error generated is shown in Fig. 7. When solar eclipse and lunar eclipse occur, their influence on the carrier phase observation can reach a half wavelength and even a whole wavelength. For phase wrapping correction model, refer to Eqs. (5)-(7) [3].

$$\boldsymbol{D} = \boldsymbol{x} - \boldsymbol{k}(\boldsymbol{k} \cdot \boldsymbol{x}) - \boldsymbol{k} \times \boldsymbol{y} \tag{5}$$

$$\overline{D} = \overline{x} - k(k \cdot \overline{x}) + k \times \overline{y}$$
(6)

$$\Delta \varphi = \operatorname{sign}(\boldsymbol{k} \cdot (\boldsymbol{\bar{D}} \times \boldsymbol{D})) \cos^{-1}(\boldsymbol{D} \cdot \boldsymbol{\bar{D}} / |\boldsymbol{D}|| \boldsymbol{\bar{D}}|)$$
(7)

where k represents the unit vector from satellite to the antenna of the receiver. (x, y, z) represents the unit vectors of the receiver antenna; $\Delta \phi$ represents the antenna phase wrapping correction; (D, \overline{D}) is the dipole vector generated between the satellite unit vector and the unit vector of receiver antenna.



Fig. 7 Three-axis directional error in non-phase wrapping correction

4.7 Earth tide correction

Perturbation celestial bodies like moon and sun have the force of gravity upon elastic earth, which causes periodical rise and fall on the earth surface. So it is called solid earth tide phenomenon. The factors that influence it include periodic term and secular bias term. Periodic term is related to diurnal term and semidiurnal term. Secular bias term is related to latitude, which prolongs the earth in the direction of the line between the earth core and perturbation celestial bodies. The line at radical direction tends to be flat.

The major influence of periodic term can be eliminated with 24 h static observation data. But the influence of secular term cannot be eliminated and it can be dealt with random walk model. The influence magnitude of solid tide is about 0.1 m, as is shown in Fig. 8. For specific correction model, refer to Eq. (8):

$$\Delta \overline{X}_{j} = \sum_{j=2}^{3} \frac{G_{Mj}}{G_{ME}} \cdot \frac{r_{E}^{4}}{\left|X_{j}\right|^{3}} \cdot \left[3l_{2} \cdot \frac{X_{p} \cdot X_{j}}{\left|X_{p}\right| \cdot \left|X_{j}\right|} \cdot \frac{X_{j}}{\left|X_{j}\right|} + \left[3\left(\frac{h_{2}}{2} - l_{2}\right) \right] \left(\frac{X_{p} \cdot X_{j}}{\left|X_{p}\right| \cdot \left|X_{j}\right|}\right)^{2} - \frac{h_{2}}{2} \right] \cdot \frac{X_{p}}{\left|X_{p}\right|} + \left[-0.025 \sin \phi \cos \phi \sin \left(\theta + \lambda\right) \right] \frac{X_{p}}{\left|X_{p}\right|}$$

$$(8)$$

where $r_{\rm E}$ represents earth radius; X_j represents the coordinate vector of perturbation celestial body (for example, moon, sun) in the geocentric reference frame;



Fig. 8 Three-axis directional error in non-solid tide correction

 $X_{\rm p}$ represents coordinate vector of the observation station in the geocentric reference frame; $G_{\rm Mj}$ represents the gravity parameter of perturbation celestial body (*j*=2 refers to the moon, *j*=3 refers to the sun); $G_{\rm ME}$ represents the earth gravity parameter; h_2 =0.6090, l_2 =0.0852; ϕ and λ are the latitude and longitude of the observation station, respectively; θ represents the uniform Greenwich sidereal time.

4.8 Ocean loading

Ocean load is similar with solid tide and is composed of diurnal term and semidiurnal term. The major difference is that it happens because of periodic rise and fall of tide. It has one magnitude smaller than solid tide in numerical value and does not have secular term. The influence of marine load is usually corrected with models. Its correction model is as follows [15]:

$$\Delta c = \sum_{j} f_j A_{cj} \cos(w_j t + \chi_j + u_j - \phi_{cj})$$
(9)

where Δc represents the influence of marine load on coordinate component of the observation station; *t* refers to time parameter; A_{cj} represents the amplitude of the influence from tide component *j* on coordinate component *c*; ϕ_{cj} represents the phase angle of the influence of tide component *j* on coordinate *c* component; f_j represents the coefficient scale of component *j*; u_j represents the phase angle deviation of component *j*; w_j represents the angular velocity of component *j*; χ_j represents the astronomy parameter of component *j*.

4.9 Atmospheric loading

Atmospheric load may result in a radical displacement of 10-25 mm on earth surface. The corresponding displacement in horizontal direction is 1/3-1/10 of radical displacement. The circle is about 14 d and is related to geographical latitude. For horizontal

and radical displacement correction models, refer to Eqs. (10) and (11) [16]:

$$u_h(r,t) = \iint q(r,r') \Delta P(r',t) G_H(\psi) \cos \varphi' d\lambda' d\varphi'$$
(10)

$$u_r(r,t) = \iint \Delta P(r',t) G_R(\psi) \cos \varphi' d\lambda' d\varphi'$$
(11)

5 Conclusions

1) The errors for precise point positioning are divided into two categories, traditional errors and endemic errors.

2) Correct all errors with precise models and make qualitative analysis of their magnitudes. In the signal transmission process, the earth rotation correction and relativistic correction surpass 1 m. Receiver antenna phase center and reference point correction, correction from satellite inertia center to antenna phase center, and solid tide correction are all between 0.1 m to 0.3 m. To ensure the accuracy of precise point positioning, correction is in urgent need. However, P1–C1 DCB correction, phase wrapping correction, marine load, and atmospheric load are less than 5 cm. In general conditions, they need no correction.

3) Precise point positioning for GNSS system should also take the differences in time references and satellite antenna phase center deviation into consideration.

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(Edited by CHEN Wei-ping)