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Rational formulation of modern geoid concept

GUO Zeng-zhang, XIAO Jian-hua
Geodesy Institute, Henan Polytechnic University, Jiaozuo 454000, China

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Abstract: Based on modern geometrical field theory, a rational interpretation of modern geodesic surface concept was formulated. The general equations for geodesic surface were given. The height concept and arc length concept were formulated strictly. Their intrinsic benefits and shortages of parameter-center reference and mass-center reference systems were studied in details. The research results show that the geodesic concept depends on the available technology. The technology-dependence feature of geodesy concept was expressed by related mathematic formulations. For the satellite-based observation system, the rationality of China-2000 Geoid Reference System was explained. Discussions about how to establish regional precision reference system were made based on general equations of arc length and space distance equations. A new correction item should be taken into consideration for arc length gauge calculation.

Key words: geodesic surface; parameter-center system; mass-center system; geodesy deformation; geoid

1 Introduction

Along with the national wide application of China-2000 Geoid System, the mass-center system is used to replace the old parameter-center system. Not only a lot of control point coordinators are modified, but also a sharp conceptual problem for engineering field researchers is raised. It is expected that a lot of arguments about how to estimate error and how to control them will be a hot-point. However, this is very shallow. The key problem and the valuable topic should be how to transform our technology to the new system and how to construct high precision local geoid system to get high accuracy. As the dynamic deformation measurements become possible in essential sense, the important step is how to perform it and how to develop the dynamic monitoring technology based on geometrical data and gravity data.

The concept of geodesic surface is based on the ideal average size and shape of the Earth. Initially, the radium of the Earth was measured by geometrical methods performed on ground surface [1]. As a natural result of sphere surface of the Earth, the first geodesic surface was constructed. After the establishment of Gauss geometry theory, the actual measurements showed that the Earth is more like an elliptic sphere [1]. Then, the elliptic geodesic surface concept was well formulated.

Along with the application of gravity measurement, the constant potential surface was used to define the geodesic surface. The geometrical definition of geodesy was replaced by the gravity field definition as a natural result of available technology application. In essential sense, the gravity height concept was the kernel of geodesic height system in elliptic sphere reference system. Based on huge gravity height data, using the ideal elliptic geodesic surface equations obtained earlier, a high precision height system was obtained. However, this advantage was achieved by modifying the geometrical coordinator system definition of elliptic sphere with the help of parameter center definition. This made the geometrical equation lost its exact theoretic formulation. To defense this modification about Gauss geometrical equations of sphere coordinators, the rationality of parameter-center system was explained again and again. No mater what reasons were given, the reality was that the gravity force direction was defined as the normal direction of elliptic surface [2]. Therefore, evaluating from available technology at that time, the parameter-center system was forced to be accepted as the result of gravity measurement technology. The significant shortage was that the height concept lost its exact geometrical meaning.

Unfortunately, as the height concept was controlled

Corresponding author: GUO Zeng-zhang; Tel/Fax: +86-391-3987666; E-mail: gzc@hpu.edu.cn

by the gravity height concept for very long time, once introducing the mass center system using the coordinators of sphere, the height concept in new geodesy system is confused by many users. If the geometrical height concept in new system is not well understood, the dynamic geodesic surface concept will be lost. This confusion will damage the related applications (such as, deformation measurement, regional high precision reference system and disaster monitoring).

This work will explain the new geodesic surface concept from three aspects: geometry, physics, measurement technology and devices. The theoretic measurement problems of regional, local dynamic deformation of geodesic surface are studied in details.

2 Geometrical field theory of geoid

Based on modern geometry theory, taking the mass-center as the original reference point, an arbitral 3-dimension coordinators can be established as (r, θ, φ). Any point within the Earth or outside the Earth can be identified by its three coordinators. Considering two near points (r, θ, φ) and (r+dr, θ+dθ, φ+dφ), the square of infinitesimal distance ds is given by Riemann geometrical equation [3] as:

$$\mathrm{d}s^2=\boldsymbol{g}_{rr}\cdot\mathrm{d}r^2+\boldsymbol{g}_{\theta\theta}\cdot\mathrm{d}\theta^2+\boldsymbol{g}_{\varphi\varphi}\cdot\mathrm{d}\varphi^2+ \\ 2\boldsymbol{g}_{r\theta}\cdot\mathrm{d}r\cdot\mathrm{d}\theta+2\boldsymbol{g}_{r\varphi}\cdot\mathrm{d}r\cdot\mathrm{d}\varphi+2\boldsymbol{g}_{\theta\varphi}\cdot\mathrm{d}\theta\cdot\mathrm{d}\varphi \tag{1}$$

where the length gauge tensor $\boldsymbol{g}_{ij}=\boldsymbol{g}_{ij}(r,\theta,\varphi)$ are functions of coordinators.

In geometrical formulation, the modern concept of the original point is a natural selection for geodesy geodesic surface which can be expressed as:

$$\boldsymbol{g}_{\theta\varphi}(r,\theta,\varphi)=0,\ \ \boldsymbol{g}_{r\theta}(r,\theta,\varphi)=0,\ \ \boldsymbol{g}_{r\varphi}(r,\theta,\varphi)=0 \tag{2}$$

This condition means that the three coordinator directions are always perpendicular to each other in any local position. It is clear that the sphere coordinator system with mass-center as the original point is a natural selection for geodesy.

The benefit is that whether in the Earth or out the Earth, the square of differential distance ds is always formulated as:

$$\mathrm{d}s^2=\boldsymbol{g}_{rr}\cdot\mathrm{d}r^2+\boldsymbol{g}_{\theta\theta}\cdot\mathrm{d}\theta^2+\boldsymbol{g}_{\varphi\varphi}\cdot\mathrm{d}\varphi^2 \tag{3}$$

This process makes the local space become conventional standard rectangular space, which is very important for the application of conventional measurement devices. This is the kernel target for geodesic surface concept.

On ground-surface, taking the sphere coordinators (θ, φ) as the natural definition of longitudinal and latitude gradients, and taking the coordinator r as the straight line distance between ground point to mass-center ($\boldsymbol{g}_{rr}(r,\theta,\varphi)=1$), the above equation is simplified as:

$$\mathrm{d}s^2=\mathrm{d}r^2+\boldsymbol{g}_{\theta\theta}\cdot\mathrm{d}\theta^2+\boldsymbol{g}_{\varphi\varphi}\cdot\mathrm{d}\varphi^2 \tag{4}$$

Taking the (θ, φ) as the intrinsic coordinators, the geodesic surface is defined by pointing a special surface (geodesic surface $R(\theta, \varphi)$) which meets the following definition equation:

$$\tilde{r}(\theta,\varphi)=R(\theta,\varphi) \tag{5}$$

How to select the special function $R(\theta, \varphi)$ depends on the choice of each nation. Most likely, it is selected by national geography features. For the global selection, the average shape of the Earth plays the main role. Once this function is selected, for two neighboring points on the geodesic surface, the differential dR is determined by the following equation:

$$\mathrm{d}R^2=\left(\frac{\partial R}{\partial\theta}\cdot\mathrm{d}\theta\right)^2+\left(\frac{\partial R}{\partial\varphi}\cdot\mathrm{d}\varphi\right)^2 \tag{6}$$

Hence, the differential distance on geodesic surface dS is formulated as:

$$\mathrm{d}S^2=\tilde{\boldsymbol{g}}_{\theta\theta}(\theta,\varphi)\cdot\mathrm{d}\theta^2+\tilde{\boldsymbol{g}}_{\varphi\varphi}(\theta,\varphi)\cdot\mathrm{d}\varphi^2 \tag{7}$$

On geodesic surface, the related components of length tensor are

$$\tilde{\boldsymbol{g}}_{\theta\theta}(\theta,\varphi)=\boldsymbol{g}_{\theta\theta}[R(\theta,\varphi),\theta,\varphi]+\left(\frac{\partial R}{\partial\theta}\right)^2 \tag{8-1}$$

$$\tilde{\boldsymbol{g}}_{\varphi\varphi}(\theta,\varphi)=\boldsymbol{g}_{\varphi\varphi}[R(\theta,\varphi),\theta,\varphi]+\left(\frac{\partial R}{\partial\varphi}\right)^2 \tag{8-2}$$

where $(\partial R/\partial\theta)^2$ and $(\partial R/\partial\varphi)^2$ are correction items caused by the local features of geodesic surface. It should be noted that the items $\boldsymbol{g}_{\theta\theta}[R(\theta, \varphi), \theta, \varphi]$ and $\boldsymbol{g}_{\varphi\varphi}[R(\theta, \varphi), \theta, \varphi]$ are exact known quantities as they are defined on sphere with local radium $R(\theta, \varphi)$. Generally speaking, the correction items can be omitted. However, in this work, they are the main topics. In fact, the basic thoughts of geodesic surface definition are to make sure the high precision of arc length on geodesic surface. The modern geodesic surface concept is to make the high precision be maintained everywhere (deep in the Earth or far away from the Earth).

On the Earth surface point $r(\theta, \varphi)$, the modern height H concept is defined as:

$$H(\theta,\varphi)=r(\theta,\varphi)-R(\theta,\varphi) \tag{9}$$

Thus, near the geodesic surface, the differential distance ds is expressed as:

$$\mathrm{d}s^2=H^2+\mathrm{d}S^2=H^2+\tilde{\boldsymbol{g}}_{\theta\theta}(\theta,\varphi)\cdot\mathrm{d}\theta^2+\tilde{\boldsymbol{g}}_{\varphi\varphi}(\theta,\varphi)\cdot\mathrm{d}\varphi^2 \tag{10}$$

By this way, the local space near ground surface is transformed as a standard rectangular space.

In other words, although on the new defined mass-center geodesic surface, the condition: $\boldsymbol{g}_{r\theta}(R, \theta, \varphi)=0$ and $\boldsymbol{g}_{r\varphi}(R, \theta, \varphi)=0$ is not exactly met, through introducing the defined Eqs. (6) and (7), high precision of Eq. (10) can be achieved. This is very clear for satellite based measurement data sets $R(\theta, \varphi)$ and $H(\theta, \varphi)$.

For a given geodesic surface $R(\theta, \varphi)$, the global components of length tensor ($\boldsymbol{g}_{\theta\theta}(\theta, \varphi)$, $\boldsymbol{g}_{\varphi\varphi}(\theta, \varphi)$) can be obtained uniquely. So, the arc length on geodesic surface can be established. Combining with the satellite height systems (defined by Eq. (9)), a high precision geodesy system is established.

Generally, the arc length coordinators (X, Y) are expressed as:

$$\mathrm{d}X=\sqrt{\tilde{\boldsymbol{g}}_{\varphi\varphi}(\theta,\varphi)}\cdot\mathrm{d}\varphi\ ,\quad \mathrm{d}Y=\sqrt{\tilde{\boldsymbol{g}}_{\theta\theta}(\theta,\varphi)}\cdot\mathrm{d}\theta \tag{11}$$

Thus, it supplies a local plane coordinator system. Although the local height direction is not exactly on the normal direction of geodesic surface plane, when the satellite height data are used, the complete equation for distance is Eq. (10). So, on abstract sense and actual engineering calculation, the local space is the standard rectangular system. The trick is that the satellite height is measured independently.

This section shows the general features of modern geodesic surface concept. Under the limited condition of measurement devices, some simplifications are required.

3 Geometrical measurement of geoid

Traditionally, the Earth is viewed as a symmetry body about pole axe. Hence, the length gauge is independent with coordinator θ. This geometrical simplification makes the Eq. (10) be simplified as:

$$\mathrm{d}s^2=H^2+\mathrm{d}S^2=H^2+\tilde{\boldsymbol{g}}_{\theta\theta}(\varphi)\cdot\mathrm{d}\theta^2+\tilde{\boldsymbol{g}}_{\varphi\varphi}(\varphi)\cdot\mathrm{d}\varphi^2 \tag{12}$$

Correspondingly, the geodesic surface is defined by equation:

$$\tilde{r}(\varphi)=R(\varphi) \tag{13}$$

Using the Earth parameters a, b (or e, $e^2=(a^2-b^2)/a^2$), the theoretic geodesic surface $R(\varphi)$ can be defined by the distance function referring to the mass-center as the function about coordinator φ.

Based on Riemann geometry, its theoretic form is expressed as:

$$R(\varphi)=\frac{1}{\sqrt{\dfrac{\cos^2\varphi}{a^2}+\dfrac{\sin^2\varphi}{a^2(1-e^2)}}} \tag{14}$$

Hence, the simple geodesic surface (roughly near the real shape of the Earth) is defined. Under this definition, the related length gauge components are

$$\tilde{\boldsymbol{g}}_{\theta\theta}(\varphi)=R^2(\varphi)\cdot\cos^2\varphi\ ,\quad \tilde{\boldsymbol{g}}_{\varphi\varphi}(\varphi)=R^2(\varphi)+\left(\frac{\mathrm{d}R}{\mathrm{d}\varphi}\right)^2 \tag{15}$$

It is clear that it is a modification of traditional geodesic surface definition.

Another simple form of geodesic surface is defined by the parameters c (pole radium) and $\alpha=(a-c)/a$ as:

$$R(\varphi)=a\cdot(1-\alpha\cdot\sin^2\varphi) \tag{16}$$

It shows that the selection of geodesic surface depends on available technology and local precision consideration. How to define the function $R(\varphi)$ is the topic for traditional geodesy theory. The mathematic research leads to the establishment of elliptic function theory.

In fact, the average shape of the Earth is not a simple analytic function $R(\varphi)$. It is constructed by measurement data based on satellite ranging. For geometrical measurement technology, taking the arc length $N(\varphi)$ and $M(\varphi)$ as the parameters, the global gauge tensor components $\boldsymbol{g}_{\theta\theta}(\varphi)$ and $\boldsymbol{g}_{\varphi\varphi}(\varphi)$ can be written as:

$$\boldsymbol{g}_{\theta\theta}(\varphi)=N^2(\varphi)\cdot\cos^2\varphi\,,$$

$$\boldsymbol{g}_{\varphi\varphi}(\varphi)=M^2(\varphi)+\left(\frac{\mathrm{d}R}{\mathrm{d}\varphi}\right)^2 \tag{17}$$

Then, the complete definition of arc length on geodesic surface $R(\varphi)$ is expressed as:

$$\mathrm{d}S^2=N^2(\varphi)\cdot\cos^2\varphi\cdot\mathrm{d}\theta^2+[M^2(\varphi)+\left(\frac{\mathrm{d}R}{\mathrm{d}\varphi}\right)^2]\cdot\mathrm{d}\varphi^2 \tag{18}$$

The height for every ground surface point is defined as:

$$H(\theta,\varphi)=r(\theta,\varphi)-R(\varphi) \tag{19}$$

Under such a definition system, the differential distance ds near geodesic surface is

$$\mathrm{d}s^2=H^2+N^2(\varphi)\cdot\cos^2\varphi\cdot\mathrm{d}\theta^2+[M^2(\varphi)+\left(\frac{\mathrm{d}R}{\mathrm{d}\varphi}\right)^2]\cdot\mathrm{d}\varphi^2 \tag{20}$$

China-2000 System (mass-center system) takes

$$N(\varphi)=\frac{a}{\sqrt{1-e^2\sin^2\varphi}}\,,$$

$$M(\varphi)=\frac{1-e^2}{1-e^2\sin^2\varphi}\cdot N(\varphi) \tag{21}$$

Hence, the geodesic surface definition is

$$R(\varphi)=\sqrt{N^2(\varphi)\cdot\cos^2\varphi+M^2(\varphi)\cdot\sin^2\varphi} \tag{22}$$

Under this geometrical definition, the local Gauss curvature (θ, φ) is expressed as:

$$K(\varphi)=\frac{1}{\sqrt{M^2(\varphi)+\left(\frac{\mathrm{d}R}{\mathrm{d}\varphi}\right)^2}\cdot N(\varphi)}\approx\frac{1}{M(\varphi)\cdot N(\varphi)} \tag{23}$$

Through triangle-check, this curvature can be evaluated.

Thus, based on exact geometrical measurements on arc length, the global gauge tensor can be determined. When the theoretic data are compared with actual measurements, the height data still can be constructed through Eq. (20).

However, as the normal of elliptic sphere is not on the gravity force direction, the height data are not measured directly in new geodesic surface definition. The rotational symmetry leads to $\boldsymbol{g}_{r\theta}(R,\theta,\varphi)=0$ on geodesic surface, but $\boldsymbol{g}_{r\varphi}(R,\theta,\varphi)\neq 0$. Hence, the real distance equation is

$$\mathrm{d}s^2=H^2+\tilde{\boldsymbol{g}}_{\theta\theta}(\varphi)\cdot\mathrm{d}\theta^2+\tilde{\boldsymbol{g}}_{\varphi\varphi}(\varphi)\cdot\mathrm{d}\varphi^2+ 2\boldsymbol{g}_{r\varphi}(\varphi)\cdot H\cdot\mathrm{d}\varphi\neq H^2+\mathrm{d}S^2 \tag{24}$$

Fortunately, on the geodesic surface, H=0. Comparing with theoretic Eq. (12), near the geodesic surface, error is not very significant even if the gravity direction is used to measure the local height. In fact, by the gauge tensor definition, theoretically, we have

$$\boldsymbol{g}_{r\varphi}=-\left|\frac{\mathrm{d}R(\varphi)}{\mathrm{d}\varphi}\right| \tag{25}$$

So, the Eq. (24) can be rewritten as:

$$\mathrm{d}s^2=H^2+\tilde{\boldsymbol{g}}_{\theta\theta}(\varphi)\cdot\mathrm{d}\theta^2+\tilde{\boldsymbol{g}}_{\varphi\varphi}(\varphi)\cdot\mathrm{d}\varphi^2- 2\cdot\left|\frac{\mathrm{d}R(\varphi)}{\mathrm{d}\varphi}\right|\cdot H\cdot\mathrm{d}\varphi \tag{26}$$

For conventional gravity based height measurement technology, in large height case, the theoretic length is shorter than the actual length. Hence, the new geodesic surface definition will limit the effectiveness of conventional height related measurements.

In modern measurements technology, as the satellite data can supply the exact data of ground surface $r(\theta, \varphi)$, after the geodesic surface radium $R(\varphi)$ is defined, the height data are measured accurately. The cost for taking this system is that when the height difference is significant, the theoretic length of Eq. (12) has significant error. However, for the modern technology based on GPS, the arc length (for remote two points) is also measured by satellite. Hence, this error will not cause intrinsic difficulty. If this essential view-point is accepted, the rationality of China-2000 Mass-center System will be supported.

In contrast, if we ignore the modern technology based on GPS technology and reasoning from the height measurement by surface geometrical technology or gravity technology, and insist on the condition $g_{r\varphi}(R,\theta,\varphi)=0$, the users will refuse the China-2000 Mass-center System and select the old parameter-center system.

4 Gravity measurement of geoid

For ground surface based measurement technology, it is difficult to determine the local direction of coordinator r of mass-center system. Hence, when the gravity direction is used as the approximation of it, the precision is limited. This error is mainly exhibited as the low accuracy of height data. To improve the height data precise, the parameter-center elliptic sphere reference system is adopted. As the gravity field plays an indispensable role in ground based technology, the parameter-center system is a natural selection.

After the invention of high precision gravity measurement devices, the geodesic surface can be measured by fitting the gravity data on a theoretic elliptic sphere surface. The geodesic surface becomes a concept of physical equal-potential surface. To distinguish from the geodesic surface determined by Gauss curvature measurements, it is referred to pseudo-geodesic surface. On this surface, using the coordinator (L, B), taking the distance to the surface along its normal direction as the local coordinator R, the local gauge tensor components are defined as:

$$\boldsymbol{g}_{LB}(R,L,B)=0,\quad \boldsymbol{g}_{RL}(R,L,B)=0,\quad \boldsymbol{g}_{RB}(R,L,B)=0 \tag{27}$$

It is this reason that makes the R coordinator not point to the sphere-center rather point to the parameter-center. If this definition is taken, the height data cannot be measured directly by satellite ranging system. In fact, in early stage of satellite application, the gravity data are measured to get the height data.

For gravity based technology, taking the local gravity direction to determine the R coordinator, using the condition Eq. (27), the gravity-dependent coordinators (L, B) are determined. Such a system is purely parameter-center system. Comparing with the mass-center system, we have

$$R\neq r,\quad L=\theta,\quad B\neq\varphi \tag{28}$$

To meet the condition Eq. (27), the parameter-center system pays a heavy cost. The natural coordinator φ and r must be modified. At the same time, the height concept is a physical concept (depends on local physical condition, usually the roughness of gravity data) rather than a pure geometrical concept. The difference between gravity height and geometrical height causes significant error. After paying such costs, the benefits are obtained. On the so-defined parameter-center system, the accurate differential distance equation is

$$\mathrm{d}s^2 = H^2 + \mathrm{d}S^2 = H^2 + \tilde{\boldsymbol{g}}_{LL}(B)\cdot \mathrm{d}L^2 + \tilde{\boldsymbol{g}}_{BB}(B)\cdot \mathrm{d}B^2 \quad (29)$$

Hence, based on gravity measurement technology, the requirements of theoretic consistence will force the users to adopt the parameter-center system to replace the mass-center system. As the early geodesic surface is originated from ideal sphere, to make the gravity technology be the main role, many theoretic reasons are made up in textbooks and papers. So, it can be imagined that once the China-2000 Mass Center System is adopted, the same reason will be the main weapon to refuse the new system.

For Xi'an parameter-center system, taking

$$N(B) = \frac{a}{\sqrt{1-e^2\sin^2 B}}, \quad M(B) = (1-e^2)\cdot N(B) \quad (30)$$

then, the mass-center system gives out the radium of geodesic surface by the following equation:

$$r(B) = \sqrt{N^2(B)\cdot[\cos^2 B + (1-e^2)\cdot \sin^2 B]} \quad (31)$$

The related components of gauge tensor are:

$$\boldsymbol{g}_{LL}(B) = N^2(B)\cdot \cos^2 B,$$

$$\boldsymbol{g}_{BB}(B) = (1-e^2)\cdot N^2(B) + \left(\frac{\mathrm{d}\tilde{r}}{\mathrm{d}\varphi}\right)^2 \quad (32)$$

However, the height in Xian System is defined by parameter-center system, $H(L,B) \neq r(L,B) - r(B)$. It is clear that the satellite ranging data $r(\theta, \varphi)$ about ground-surface cannot be directly used to get the results in Xian System. In fact, this difference is the main source for errors.

Furthermore, as the gravity force depends on the mass distribution within the Earth, its random distribution will make the height data (based on gravity) be not the exact feature of locality. The global fitting and local roughness are always the painstaking contradict for parameter-center system determination [4].

Therefore, it should have no difficulty to understand that for GPS data based measurement technology, the mass-center system rather than the parameter-center system must be used.

5 Space measurement of geoid

In early stage, when the ground gravity data show a lot of ripples, the data fitting results are not very good. On theoretic consideration, far away from the ground, the roughness can be significantly reduced and make a better fitting result. Hence, initially, the satellite is used to supply the gravity data. Then, the data are used to deduce the geoid surface data.

After the laser distance measurement is used in satellite networks, the huge ranging data are coordinated by the mass-center spherical coordinators. If the geoid system is a parameter-center system, the ground based measurement data and the space based measurement data will be in a state of logic inconsistency. Relatively, the space based measurements of the Earth surface $r(\theta, \varphi)$ should be the reliable data.

Based on this confidence, the natural way is to estimate the average surface of the Earth to define a suitable surface $R(\theta, \varphi)$ as the geoid. Then, the height data are purely geometrical and can be obtained directly with the same accuracy as $H(\theta, \varphi)$. As the height measurement technology is changed, the ground surface measurement technology of height is abounded. Based on the given geoid surface function $R(\theta, \varphi)$, after calculating its partial derivatives about its two coordinators, Eq. (7) defines the differential distance equation. On this point, the local high precision geoid can be established with the same accuracy everywhere. Its error only depends on the satellite data and the theoretic calculation process. This is important because it removes the local physical condition dependence for local geoid construction. Hence, the local geoid and the global geoid have the same degree of accuracy.

Additionally, independent arc length measurements can be used to check the accuracy of Eqs. (8-1) and (8-2). So, the mass-center system selection is natural. Then, a geoid theory based on mass-center system and modern geometry can be constructed.

The conclusion is that China-2000 Geoid System is the natural results of satellite technology development.

In fact, once the connecting-network between ground based networks and space based networks is fully constructed, several new geoids (reference surfaces) faraway from ground may be constructed. In that case, the consistence among geoids is a key factor for error control technology. Only after the measurement accuracy is high enough, to measure the deformation and dynamic features of the Earth (including internal region) can be achieved, as it is predicted by the dynamic geoid concept [5].

Under such a space based measurement stage, constructing high precision local geoid which is purely geometrical and modernization becomes a reality. For this local geoid, the old parameter-center elliptic geoid concept and rotational symmetry should be abandoned. The local geoid should based on local actual condition and geography. For this target, the equations given in this work will be very helpful.

Although the global geoid system may significantly differ from local geoid, as the common source of data are satellite based $r(\theta, \varphi)$, their intrinsic relations are maintained exactly. Hence, the global controlling data can also be used to check the local geoid accuracy rather only limited to check the global geoid accuracy. The consistency between the global geoid system and local geoid system is maintained. This consistency is not possible for parameter-center geoid system.

On the other hand, the equal-gravity surface is very important for local engineering application, especially for water related system and foundation constructions. The best way is to construct local gravity height system by the similar way. That is to say: 1) define the equal-gravity potential surface: $\widetilde{R}(\theta,\varphi)$; and 2) define the gravity height: $\widetilde{H}(\theta,\varphi)=r(\theta,\varphi)-\widetilde{R}(\theta,\varphi)$.

Under such a definition, the difference (and its variation about time) between the geometrical height and the gravity height becomes the essential data to study the internal motion of materials within the Earth. This data will be very important for environment related researches. In theoretic sense, the geometrical geoid and the gravity geoid (both based on the same mass-center system) will become the contents of geodesy technology for Earth internal region.

To identify the variation between the geometrical geoid and gravity geoid, a deep-space reference system should be used to get the average variation about time. So, both of geoids are based on dynamic concept [6−7].

In fact, the crust deformation measurements will be promoted greatly by the new geoid definition [8−9].

For geodesy related engineering, local deformation is the main topic. How to improve accuracy based on new geoid system data and the old geoid system data is a challenge job. Without deep theoretic understanding about the geoid system concept, the new application (dynamic measurements) is not possible.

6 Conclusions

1) Although the height direction is taken as the radium coordinator direction (originated from mass-center point) and it is no longer the normal direction of geodesic surface, the local space near ground surface still is a standard rectangular space. Hence, the conventional differential distance equation is invariant in form.

2) Once a special surface $r=R(\theta, \varphi)$ is taken as the geodesic surface, the arc length gauges along θ and φ directions must be supplied a correction item. However, the conventional form as a first approximation is maintained.

3) For China-2000 Geoid System, the arc length distance differential equations on geodesic surface are given. The correction items should be taken in consideration for high precision measurements.

4) The height concept in new system depends on the geometrical feature of geodesic surface, rather than depends on the physical feature of geodesic surface (the local curvature determined by gravity field). Hence, the new geodesic surface definition will limit the effectiveness of conventional height related measurements (ground-based).

5) A new correction item should be taken into consideration for arc length gauge calculation.

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